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PIEZOELECTRIC INSTRUMENTS OF HIGH NATURAL FREQUENCY
VIBRATION CHARACTERISTICS AND PROTECTION
AGAINST INTERFERENCE BY MASS FORCES

By Werner Gohlke

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I. SUMMARY

The exploration of the processes accompanying engine combustion demands quick-responding pressure-recording instruments, among which the piezoelectric type has found widespread use because of its especially propitious properties as vibration-recording instruments for high frequencies. Lacking appropriate test methods, the potential errors of piezoelectric recorders in dynamic measurements could only be estimated up to now.

In the present report a test method is described by means of which the resonance curves of the piezoelectric pickup can be determined; hence an instrumental appraisal of the vibration characteristics of piezoelectric recorders is obtainable.

II. THE PIEZOELECTRIC EFFECT

The conventional piezo instruments make use of the direct piezoelectric effect according to which electric charges are developed on certain surfaces of certain crystals when subjected to strain. The electric charge is proportional to the force P , that is, the volume of charge being

$$Q = d P \quad (1)$$

* "Quarzdruckmessgeräte hoher Eigenfrequenz Schwingungseigenschaften und Abhilfe gegen die Störung durch Massenkräfte." VDI-Forschungsheft 407, XII Bd., March-April 1941, pp. 1-25.

where d is the piezoelectric modulus expressed in kilograms. It is dependent upon the type of crystal and the direction of the force relative to the axes of the crystal. For pressure measurements quartz crystal is commonly used because of its mechanical strength and durability. The block or plate is cut from a quartz crystal as indicated in figure 1 so that the strain along the x -axis acts on a y,z -surface where the charge is measured; x is called the electric, y the neutral, and z the optical axis. The strain could also be developed along the y -axis and the charge measured on the y,z -surface, but this arrangement has found little use so far for pressure measurements. Since quantity d depends on the direction of cut of the crystal plate, numerical data must carry d with the appropriate subscripts. In x -direction the piezoelectric modulus of quartz is, in centimeter-gram-second units,

$$d = d_{11} = 6.36 \times 10^{-8} \text{ cm}^{1/2} \text{g}^{-1/2} \text{ s}$$

in engineering units of measurement

$$d = d_{11} = 2.03 \times 10^{-11} \text{ C/kg}$$

or

$$d = d_{11} = 2.12 \times 10^{-10} \text{ cm/V}$$

The charge developed on the surface at right angles to the electric axis charges the capacity associated with these surfaces to a correspondent voltage, which is then measured with a suitable instrument.

Although the piezoelectric effect has been known since the last century, its use for pressure recording had never been seriously attempted before 1930.

III. APPRAISAL OF PERFORMANCE OF A PIEZOELECTRIC TEST SETUP

A piezoelectric setup for recording alternating pressures ordinarily comprises

- (1) Piezoelectric pickup
- (2) Amplifier

(3) Indicating instrument

(4) Interconnecting lines

(5) Power supply

All these arrangements can influence the reproduction of the test quantity especially at high frequencies, hence have a frequency relationship if incorrectly designed. The present report deals mainly with the vibration characteristics of the pressure pickup and secondarily with the other parts. With this in mind we begin with items 2 to 5 regarding their effects on the reproduction of the true pressure variation at high frequencies, whereby it is recommended to proceed from the reading instrument and work back toward the piezoelectric pickup.

1. Reading Instruments

These instruments are either loop oscillographs or cathode-ray oscillographs.

Loop oscillographs. - The reading element is a loop rotatably mounted in a magnet field. In static measurements its deflections are largely proportional to the intensity of the current passing through the loop. The natural frequency of the loops is governed by the mass and the resetting force as well as by the damping of the system afforded by the oil-filled loop housing. Since this natural frequency ordinarily cannot be changed except by special auxiliary means, it is necessary to fall back on the loops with defined vibration characteristics as supplied by the manufacturer. The only loop types of high natural frequency suitable for rapid pressure variations unfortunately require a comparatively high current, that is, they are fairly insensitive because of the great resetting forces necessary for high natural frequency. Table 1 gives the data of the Siemens-Halske types of loop.

TABLE I.-- DATA OF LOOPS FOR SIEMENS & HALSKE UNIVERSAL OSCILLOGRAPH

Type	Natural frequency f_0 in air (Hz)	Maximum deflection (mm)	Maximum cur- rent strength (ma)	Sensitivity (mm/ma)
8	1,200	± 80	1.5	45
5	2,000	± 80	6	15.5
4	3,000	± 80	20	4.2
1	5,500	± 80	100	.83
2	10,000	± 80	280	.29
7	20,000	± 15	250	.057

Type 2 has a susceptibility of 0.29 millimeter per milliampere. An acceptable diagram height of about 60 millimeters requires about a 200-milliampere loop current. The natural frequency of the undamped loop is given at around 10,000 Hz. Damping lowers it to around 4500 Hz. The next higher frequency is that of type 7 with about 20,000 Hz. With a maximum current of 250 milliamperes its deflection is only about 14.5 millimeters. At this deflection there is a considerable inaccuracy in the heavy trace made by the light beam. Because of the diagram height the 10,000 Hz loop will be preferable, especially since the production of the required loop currents with amplifiers itself becomes inconvenient. The recorded frequency variation of this type of loop is shown in figure 2 for three brand-new samples a, b, and c. The error in amplitude is already considerable at one-third of the natural frequency and far in excess of the instrumental errors commonly associated with static measurements of this kind.

Cathode-ray oscillographs.— Here a differentiation must be made between the so-called electronic-ray oscillograph and the high-capacity oscillograph. On the former, the trace from the fluorescent screen of the tube is recorded on a photographic film held on the outside; on the latter, the trace from the electronic ray can be recorded directly on the photographic film located in the evacuated oscillograph tube. The former is very appropriate for reproducing high frequencies when dealing with periodic processes the traces of which can be recorded over one another on the fluorescent screen as often as desired. Both types are free from inertia for long-period processes compared to the electron variation interval between the deflection plates when high-vacuum tubes are used, as

presumed here. The first type permits recording speeds up to 1000 meters per second. Special valves with high anode voltages, such as after-acceleration valves in conjunction with suitable optics are practical even at 10 kilometers per second. The Allgemeine Elektrizitäts-Gesellschaft thus quotes 50 kilometers per second for its tube with special optics, the trace on the fluorescent screen being much reduced. Since the thickness of the trace decreases at the same scale as the picture, it can be subsequently magnified again. On the other hand, the high anode voltages required for greater beam intensity cause a greater wear on the fluorescent screen of the tube and hence reduce the life of the tube. Although much higher recording speeds (up to 10^4 km/sec) have been reached, the writer has no knowledge of the life of the tubes.

For a sinusoidal process of n complete oscillations, that is, to be recorded with a diagram height $2x_0$ and a diagram width b proportional to time, and the frequency of which is f , the maximum speed of the light spot amounts to

$$v_s = f \sqrt{(2 \pi x_0)^2 + \left(\frac{b}{n}\right)^2} \quad (2)$$

For an electronic ray tube with a recording speed of 1000 meters per second and a chart size of 50 by 50 square millimeters, the precise frequency recordable at $n = 1$ is:

$$f = 6070 \text{ Hz}$$

Sinusoidal processes of higher frequency are therefore impossible to reproduce by such a tube with a 50 millimeter diagram height, which is all the more true as the screen is variously illuminated by scattered electrons, so that the blackening of the receiving material is not solely due to the recording electronic ray. For rough estimates the second term below the root representing the speed of the focal point in direction of the time axis can be discounted, so that

$$v_s = x_0 \omega$$

with ω the natural frequency.

Distortions also occur toward the screen edges, which become especially noticeable on multiple-ray tubes because of eccentric disposition of the different beam systems. In addition, high vacuum tubes should be operated exclusively with deflection voltages that are symmetrical to the ground potential, otherwise the reading errors become very great. Figure 3 illustrates the distortion of the coordinates of the electronic-ray tube of a commercial engine indicator which, despite the use of a high-vacuum electronic-ray tube, operates with unsymmetrical deflection voltages.

On the high-capacity cathode-ray oscillographs with recording speeds up to 30,000 kilometers per second (one-tenth of the speed of light) the distortions are easier to avoid, though it involves considerable outlay.

The cathode-ray oscillographs operate at voltages of the order of magnitude of 200 volts, which are easier to obtain with amplifiers than the high-voltage intensities for loop oscillographs.

As regards the reading instruments, it may be briefly stated that loop oscillographs afford sufficiently large diagrams with certain amplitude and phase errors at frequencies up to about 10^3 Hz, while electronic-ray tubes also are satisfactory up to 10^4 Hz with the degree of accuracy customary for such tubes, although the life of the tubes is comparatively short (several hundred hours). For special short-time processes the high-capacity oscillograph with incorporated recording material is available.

2. Amplifier and Current Supply

On none of the cited reading instruments can the output be controlled by a pressure quartz without amplification at pressures such as are encountered in internal-combustion engines. The amplifier must first reproduce processes from 0 to 10^4 Hz - in special cases even higher - correct in amplitude and phase, and second, possess an input resistance such as to prevent the pressure proportional charges delivered by the piezoelectric pickup from flowing off.

The first requirement calls for a direct current amplifier which must indicate no frequency variation up to 10^4 Hz (or even higher). The direct-current amplifier is

a resistance amplifier with the separate stages in pure resistance coupling. Omission of the coupling condensers on multistage amplifiers causes difficulties in the input and inconstancy in the operating voltages. The frequency variation in a resistance amplifier is due to the unavoidable valve and control capacities being placed parallel to the resistances, which lower the impedance in increasing measure with rising frequency. Since the amplification depends upon the size of the anode resistance, the amplification factor drops with rising frequency. In consequence the anode resistances are made so small from the very beginning that the harmful capacitances do not become effective except at sufficiently high frequencies. This in turn lowers the amplification per stage, hence requires more stages.

The second requirement of high-input resistance can be met by an electrometer tube, such as the AEG type T 114, the amplification factor of which is, at the most, equal to 1 for infinitely great outside resistance, but, since the operation is with low-resistance values, it is substantially less.

Amplifier for loop oscillographs. - The 10,000-Hz loop demands from the amplifier an output control range of about 200 milliamperes in the last stage. These comparatively high-current intensities are usually attempted with standard broadcasting tubes, especially the triode the output of which, with two tubes connected in parallel and suitable outside resistance, can be controlled fairly linearly up to about 200 amperes. The input voltage variation of the last stage must then amount to about 30 volts. At 0.5 to 1 volt measuring effect on the piezo-electric pickup, 30 to 60 time intermediate amplification would be necessary. Since the input tube, as stated, does not only amplify but weaken the measuring effect, a higher amplification by a voltage amplifier stage between the input tube and the last stage is necessary. The result is a three-stage direct-current amplifier. But such an apparatus can hardly be fed from an apparatus connected to the rectifier power supply because of the requirement of maximum-voltage constancy and the fact that equipment working from the rectifier power supply with or without stabilization possesses considerable internal resistances which are not constant in the desired frequency range and far from negligible even at low plate resistances. This applies particularly to the last stage which has a particularly low anode resistance or else none at all.

Acceptable measurements must therefore be made with plate batteries which, in conjunction with loop types of high natural oscillation frequency, must supply 200 to 300 milliamperes for longer periods with sufficient constancy at a voltage of around 300 volts.

Amplifier for cathode-ray oscillographs.- Here the amplification conditions are more favorable. To obtain a 200-volt deflection voltage requires about 200 to 400 time amplification. Here also three stages are necessary in many cases if a clear reproduction at high frequencies is appreciated. Since the last stage, then, has to supply high voltage rather than high current intensity, the plate resistances are permitted to be so high that the frequency dependence of the internal resistance from power supplies which usually are stabilized can be disregarded. However, in view of the frequency variation, the plate resistance of the last stage must not exceed a certain value, because parallel to it are located the capacitance of the deflection plates as well as of the interconnecting lines between amplifier and cathode-ray tube. Inasmuch as only high-vacuum cathode-ray tubes are to be used, the output of this amplifier must be fitted with a counterbalanced or push-pull control capable of supplying the voltage symmetrical to the ground even by direct-current amplification. The amplification difficulties are correspondingly greater if frequencies higher than 10^4 Hz are to be correctly reproduced.

To sum up: given ample tubes and current supply, amplifiers suitable for operating loop or cathode-ray oscillographs are obtainable.

3. Pressure Pickup Input

The weakest spot of the piezoelectric test method is that theoretically charges must be measured. For this reason the afore-mentioned highly insulated input electrometer tube and the perfect insulation of all areas of the line between pickup and magnifier are dictated. Vibrations such as those caused by a running motor are not permitted to produce test errors. This is essential especially in low-pressure measurements, because the capacitance on the input grid must be small in order to achieve sufficiently high voltages. The vibrations cause, apart from capacity changes, much more seriously disturbing charges due to frictional electricity. However, special cables are now commercially available by means of which these two sources of error can be largely avoided.

These sources of error on reading instruments, amplifiers, current supply and lines, while known in general outline, are usually not sufficiently appreciated on commercial equipment, as indicated in figure 4.

4. Piezoelectric Pickup

Very little attention has been paid, heretofore, to the vibration characteristics of the piezoelectric pickup itself. The reason lies in the lack of suitable investigating methods.

To be able to use the quartz crystal for gas pressure measurements, it must be provided with a frame or casing fitted for the purpose in hand. For the recording of combustion pressures in engines, standardized casing forms have been developed to some extent, since the pressure measurement has to be made on the spark-plug hole of the engine cylinder. The gas pressure, however, is transmitted by various means to the quartz, whereby the following facts must be borne in mind:

(1) The quartz must be protected from the hot combustion gases, since its properties are to a certain extent dependent on the temperature.

(2) The transmission links should be of minimum weight and act like very stiff springs of linear characteristics in order to assure a straight characteristic and high natural frequency from the test arrangement.

(3) The transmission links should maintain their characteristics even during prolonged operation and not be susceptible to temperature fluctuations.

(4) The reading should be independent of the type of restraint on the pressure pickup by the engine.

(5) Vibrations should not affect the test result.

The conventional types of piezoelectric pickup represent a compromise between partially contradictory requirements. In general, the quartz plates are disposed some distance from the combustion chamber so that the gas passage or a piston can be used as pressure transmission medium. A gas passage has the particular drawback to the extent that for a sufficiently steep impulse

it gives rise to natural oscillations which falsify the results. In some design types, such as those indicated in figure 5, the gas passage is therefore divided in several narrow channels for damping the natural oscillations of the passage. But this results in a Helmholtz resonator of comparatively low natural frequency. It amounts to

$$\omega_0 = 2 \pi f_0 = c \sqrt{\frac{F}{Vl}} \quad (3)$$

for the pulsation at approximately constant air density.

Here

F section of the channels

l channel length

V volume of resonator space

c = 330 meters per second, the velocity of sound at 20° C

For the reproduced type it approximates to:

F = 7 x 0.049 square centimeter = 0.34 square centimeter

V = 0.8 cubic centimeter

l = 1.7 centimeters

whence

$\omega_0 = 16,500 \text{ seconds}^{-1}$

$f_0 = 2630 \text{ Hz}$

for air in normal state. Alternating pressures of frequencies of this order of magnitude will no longer be able to react correctly on the quartz plates of the pickup.

The first natural oscillation of the pickup determined by the transmission channel itself is located at 2630 Hz, and hence limits the practicability of the pickup for higher frequencies.

Filling the passage with a piston or plunger, which for reasons of weight-saving was manufactured as a hollow body, makes the mass before the quartz plates comparatively great, hence lowers the natural frequency.

Placing the quartz itself in the threaded hole is more beneficial but, since the spark-plug thread usually runs to 14 by 1.25 millimeters, the quartz plates and the pressure surface of the pickup become very small, hence its sensitivity will be less also. In addition, it is difficult to install a practical water-cooling system.

In order to keep the elastic travel of the quartz column with the correlated intermediate pieces and other transmission links small and secure straight spring characteristics, all contact surfaces must be carefully fitted. To achieve a perfect contact of these surfaces, a certain amount of pre-tension is required even with the best polish. The pre-tension is furnished by a flat spring or spring bushing. The advantages of the latter were proved by Meurer in static tests, so that in most cases the use of flat springs has now been abandoned. The piezoelectric pickup described by Meurer is shown somewhat modified in figure 6.

As late as 1938, the literature ascribed natural frequencies of about 10^5 Hz to the then-known pressure pickups. But gradually more caution was used in the treatment of natural oscillations. There also has been lack of attempts to determine the natural oscillations of piezoelectric pickups by excitation, which can succeed only under certain conditions, which usually were not correctly known and therefore could not be kept. We find statements such as "the natural oscillation of the employed piezoelectric pickup was located so high that it could not be excited even with a hammer." As may be shown, natural oscillations can only be sufficiently excited by strict observance of a certain mass ratio between excited and exciting mass. Moreover, it remains with the aforementioned characteristics of the remaining test arrangement whether the comparatively high natural oscillation frequency, even when it has been started, can be amplified and traced. The intensity of the excitation of natural oscillations is governed by the oscillation energy E_s transmitted by impact excitation on to the system to be excited. Under simplifying assumptions it can be computed that the ratio m_1/m_2 of exciting to excited mass of the oscillator defines the oscillation energy transmitted to

the oscillating system, as figure 7 indicates. It is found that small masses are beneficial for the impact excitation, which does not occur when a hammer is used. But to attempt to ascertain the resonance curve and hence the frequency variation of the pressure pickup from the complex free oscillation curve, as exemplified in figure 8, is very difficult, since it is, in general, a system of many degrees of freedom with a great many natural frequencies.

It is generally conceded that the resonance curve defines the frequency limits within which the test instrument can be used with satisfactory accuracy. The knowledge of the vibration characteristics is therefore of decisive importance for the use of the test method. Accordingly the aim of the subsequent study will be to denote the vibration characteristics of piezoelectric pickups by their resonance curves and to deduce therefrom the rules for the design of piezoelectric pickups.

IV. METHOD OF RECORDING OF RESONANCE

CURVES OF PIEZOELECTRIC PICKUPS

A method of producing alternating pressures of traceable constant amplitude (several atm, say) and any frequency of the order of magnitude of 10^4 Hz is unknown. The direct determination of the resonance curves of piezoelectric pickups from the alternating stresses on the quartz under the effect of alternating pressures of constant amplitude and variable frequency was impossible. An attempt was therefore made to use the body acoustics to define the frequency variation; but it is difficult to produce body acoustics of sufficiently great intensity at high frequencies. Besides, coupling phenomena between transmitter, transmission links, and pressure pickup vitiate the results. The electric excitation of the quartz plates by means of the reciprocal piezoelectric effect offered a way out.

1. The Reciprocal Piezoelectric Effect

If a piezoelectric crystal is subjected to an electric field that produces the stress U on the corresponding crystal surfaces, the crystal deforms. If the field has

the direction of the x-axis (electric axis) the strain in the direction of the x-axis independent of the size of the quartz plate is

$$u = d_{11} U \quad (4)$$

(For quartz it is $d_{11} = 2.12 \times 10^{-10}$ cm/volt.)

At the same time, the crystal is strained in the direction of the y-axis by an amount

$$v = - d_{11} U \frac{F_x}{F_y} \quad (5)$$

where F_x is the quartz surface orientated at right angles to the x-axis, F_y at right angles to the y-axis. A clear explanation of the piezoelectric effects has been given by Meissner. The previously electrically neutral crystal cell is polarized by elastic crushing of the crystal lattice. Inversely, an electric field in a piezoelectric crystal not only displaces the building stones of the lattice but also changes its dimensions. The strain of the crystal does not increase exactly linear with the applied stress. But, since departure from linearity is not appreciable except at much higher exciting voltages than employed here, this can be completely disregarded in the present instance.

2. Theoretical Considerations

The theoretical principles of the test method are exemplified on the piezoelectric pickup, sketched in figure 9,

where

- c_1 elasticity of the casing mass of the pickup relative to a mass of assumedly infinite size (kg/cm)
- k_1 damping of system (m_1, c_1) , (kgs/cm)
- m_1 mass of pressure pickup casing (kgs²/cm)
- c_2 elasticity of quartz column and of pre-tension device, as of the spring bushing (kg/cm)

- k_2 damping of system (m_2, c_2)
 m_2 mass disposed before quartz, as of pressure transmitting piston
 μ_{21} mass ratio $\frac{m_2}{m_1}$
 f frequency of exciter forces (Hz)
 ω pulsation of exciter forces (s^{-1})
 $f_1 = \frac{1}{2\pi} \sqrt{\frac{c_1}{m_1}}$
 $f_2 = \frac{1}{2\pi} \sqrt{\frac{c_2}{m_2}}$
 x_1, x_2 deflections of mass m_1 and m_2 , positive downward (cm)
 P_0 amplitude of force (kg)

Under normal use of the pressure pickup as pressure instrument, the forces acting on the lowest surface, that is, on mass m_2 , are the constant force of the pre-tension and the alternating force $P_0 \sin \omega t$. The reading consists of the length change $x_2 - x_1$ of the quartz column c_2 . The electric excitation of the quartz produces between m_1 and m_2 an alternating force which periodically lengthens and shortens spring c_2 . Now the resonance curve of the pickup traversed by the charge amplitude on the quartz surfaces under variable frequency of the alternating gas pressure can be put equal under certain assumptions to that covered by the amplitude x_2 of the bottom motion of the pickup by electric excitation of the quartzes.

Assuming damping proportional to speed while discounting gravity, the admission of mass m_2 with alternating gas pressure follows the law

$$m_1 \frac{d^2 x_1}{dt^2} + c_1 x_1 + k_1 \frac{dx_1}{dt} + c_2 (x_1 - x_2) + k_2 \left(\frac{dx_1}{dt} - \frac{dx_2}{dt} \right) = 0 \quad (6)$$

$$m_2 \frac{d^2 x_2}{dt^2} + c_2 (x_2 - x_1) + k_2 \left(\frac{dx_2}{dt} - \frac{dx_1}{dt} \right) = -P_0 \sin \omega t \quad (7)$$

The transitory process is not taken into account since the transitory state defines the resonance curve. The damping forces are neglected for simplification. Then we can write:

$$-m_1 \omega^2 x_1 + c_1 x_1 + c_2 (x_1 - x_2) = 0 \quad (8)$$

$$-m_2 \omega^2 x_2 + c_2 (x_2 - x_1) = -P_0 \quad (9)$$

The deflection x_1/P_0 referred to unit force is

$$\frac{x_1}{P_0} = - \frac{c_2}{\omega^2 (m_1 m_2 \omega^2 - c_2 m_1 - c_1 m_2 - c_2 m_2) + c_1 c_2} \quad (10)$$

and

$$\frac{x_2}{P_0} = \frac{1}{m_2 \omega^2 - c_2} \left(1 - \frac{c_2 x_1}{P_0} \right) \quad (11)$$

whence, after several changes:

$$\frac{x_2 - x_1}{P_0} = \frac{1}{m_2 \omega^2 \left[1 - \left(\frac{f_2}{f} \right)^2 \right]} \left(1 + \frac{\mu_{21} \left(\frac{f_2}{f} \right)^2}{\left[1 - \left(\frac{f_1}{f} \right)^2 \right] \left[1 - \left(\frac{f_2}{f} \right)^2 \right] - \mu_{21} \left(\frac{f_2}{f} \right)^2} \right) \quad (12)$$

By electric excitation of the quartz plates it gives:

$$-m_1 \omega^2 x_1 + c_1 x_1 + j\omega k_1 x_1 + c_2 (x_1 - x_2) + j\omega k_2 (x_1 - x_2) = P_0 \quad (13)$$

$$-m_2 \omega^2 x_2 + c_2 (x_2 - x_1) + j\omega k_2 (x_2 - x_1) = -P_0 \quad (14)$$

whence

$$\frac{x_1}{P_0} = \frac{-m_2 \omega^2 [\omega^2 (m_1 m_2 \omega^2 - A) + c_1 c_2] + j\omega^3 m_2 [-\omega^2 B + D]}{[\omega^2 (m_1 m_2 \omega^2 - A) + c_1 c_2]^2 + \omega^2 [-\omega^2 B + D]^2} \quad (15)$$

where

$$A = m_1 c_2 + m_2 c_1 + m_2 c_2 + k_1 k_2 \quad (\text{kg}^2 \text{s}^2 / \text{cm}^2)$$

$$B = m_1 k_2 + m_2 k_1 + m_2 k_2 \quad (\text{kg}^2 \text{s}^3 / \text{cm}^2)$$

$$D = k_1 c_2 + k_2 c_1 \quad (\text{kg}^2 \text{s} / \text{cm}^2)$$

The deflection of the bottom surface determined by the test method is

$$\frac{x_2}{P_0} = \frac{(m_2 \omega^2 - c_2) + j \omega k_2}{(m_2 \omega^2 - c_2)^2 + \omega^2 k_2^2} + \frac{x_1}{P_0} \frac{\omega^2 (k_2^2 - m_2 c_2) + c_2^2 - j m_2 \omega^3 k_2}{(m_2 \omega^2 - c_2)^2 + \omega^2 k_2^2} \quad (16)$$

whence, neglecting the damping forces again,

$$\frac{x_2}{P_0} = \frac{1}{m_2 \omega^2 \left[1 - \left(\frac{f_2}{f} \right)^2 \right]} \left(1 + \frac{\mu_{21} \left(\frac{f_2}{f} \right)^2}{\left[1 - \left(\frac{f_1}{f} \right)^2 \right] \left[1 - \left(\frac{f_2}{f} \right)^2 \right] - \mu_{21} \left(\frac{f_2}{f} \right)^2} \right) \quad (17)$$

or the same expression as obtained for $(x_2 - x_1)/P_0$ by admission of mass m_2 with alternating gas pressure. Hence the desired resonance curve can be obtained by electrical excitation of the quartz plates and subsequent measurement of x_2 . This holds true under the assumption of small damping forces.

The resonance points for $(x_2 - x_1)/P_0$ follow from the relation

$$f^4 - f^2 [(1 + \mu_{21}) f_2^2 + f_1^2] + f_1^2 f_2^2 = 0 \quad (18)$$

after equating the denominator in the brackets of equation (12) to zero.

Then

$$f^2 = \frac{1}{2} [(1 + \mu_{21}) f_2^2 + f_1^2] \pm \sqrt{\frac{1}{4} [(1 + \mu_{21}) f_2^2 + f_1^2]^2 - f_1^2 f_2^2} \quad (19)$$

For purposes of illustration figure 10 gives several resonance curves computed according to equation (12), the

required constants of the oscillation equation being assumed. The graph brings out the effect of the distortion term contained in the bracketed term of equation (12). For the small mass ratio $\mu_{21} = 0.01$ of case b the resonance curve of the coupled system is practically in agreement with that of the simple system (m_2, c_2) , except for the very narrow frequency range in vicinity of 500 Hz, where it is changed by the resonance of the system (m_1, c_1) . But for mass ratio $\mu_{21} = 0.25$ of case a the conditions are otherwise.

It may therefore be assumed that for small mass ratios the distortion will be very low even for comparatively little damping. This was, in fact, borne out in the subsequent studies (fig. 38). It proves the advantage of a small mass ratio which is best secured by a reduction of the mass m_2 placed before the quartz plates.

3. Measurements with Sound Pressure Meter

The excitation of piezoelectric pickups with electric alternating voltage had been attempted elsewhere without success. In particular, there was a lack of test methods for proving the resonance areas. In the present instance, we succeeded in getting an audible tone by exciting a piezoelectric pickup with alternating voltage of audio-frequency. Although very weak, it could still be measured with a Siemens-Halske sound pressure meter. With appropriate test equipment distinct resonance points were traceable which permitted a record - even if not very plain - of the resonance curve.

With this in view the tests, illustrated in figure 11, were undertaken. The quartz plates a of the piezoelectric pickup D are excited by buzzer S by progressively varying the frequency f but keeping the voltage constant, causing the pickup to forced oscillations in x-direction. The sound pressure p produced hereby charges the condenser microphone KM of the sound pressure meter SM. Because of the low sound intensity the distance between D and KM had to be kept small - between 5.6 and 40 centimeters.

The experimental chamber had normal room walls and were not free from reflection. Resonance curves recorded by this method on the pickup of figure 12 are shown in figures 13 and 14. In figure 13 the pickup was mounted on a soft base facing the condenser microphone. It afforded only two distinct resonance areas.

To simulate the actual conditions on an engine the pickup of figure 14 was screwed into a 133- by 175- by 20-cubic millimeter steel plate. Then the total assembly was placed in a box padded with glass wadding in order to minimize the reflection capacity of the walls. The radiation is considerably increased by the steel plate so that even smaller departures from the initial deflection could be detected. The first resonance peak, very little damped, occurs at 3150 Hz. Further, more vigorously damped peaks follow in direction of rising frequency, the position of which varies, however, with the distance between pickup and receiver. Here the length of the air column between pickup and microphone seems to be of some influence. The next resonance peak, which is high, weakly damped and independent of the distance x between pickup and microphone, occurs at about 11,500 Hz. It is situated at approximately the same place as before in figure 13. The resonance points of the pickup could be accurately enough ascertained by varying the screwed-on mass and the distance, but as the use of condenser microphones stops at about 10,000 Hz, the higher frequencies obtained involved either the design of a new microphone or else a modification of the method. In the first instance, the interference tendency of acoustic measurements would remain.

4. Improved Method

The oscillation amplitude of the pickup bottom is outside of the resonance of the order of magnitude of 10^6 millimeters = 10 Å. It is smaller by about 3 powers of tens than the wave length of visible light so that stroboscopic interference methods with visible light, for instance, would no longer suffice for the measurements. In consequence we retained the principle of the test method while modifying the test element.

Calculation of measuring effect.- In the new version indicated in figure 15, the bottom surface b of the pickup faces a flat electrode a at distance d so that both surfaces form the poles or sides of a condenser C . This condenser is charged across the resistance R by a battery with voltage E . Moving the bottom surface by Δx in direction x varies the capacitance C . The variation in charge produced hereby induces a current through resistance R which causes a voltage drop at R . In this manner a periodic distance variation produces a periodic alternating voltage on R which is fed into the grid of an amplifier tube.

For the calculation the capacitance variation is assumed to follow the law

$$C = C_0 + C_1 \sin \omega t \quad (20)$$

whence, according to figure 16

$$E - i R = \frac{1}{C} \int i \, dt \quad (21)$$

and

$$(E - i R) (C_0 + C_1 \sin \omega t) = \int i \, dt \quad (22)$$

Differentiation with respect to t affords

$$(C_0 + C_1 \sin \omega t) R \frac{di}{dt} + (1 + R \omega C_1 \cos \omega t) i - E \omega C_1 \cos \omega t = 0 \quad (23)$$

With the Fourier series

$$i = I_1 \sin (\omega t + \varphi_1) + I_2 \sin (2\omega t + \varphi_2) + \dots \quad (24)$$

for i as function of t , the coefficients are:

$$I_1 = \frac{C_1}{C_0} \frac{E}{\sqrt{R^2 + \left(\frac{1}{\omega C_0}\right)^2}} \quad (25)$$

and

$$I_2 = - \left(\frac{C_1}{C_0}\right)^2 \frac{ER}{\sqrt{\left[R^2 + \left(\frac{1}{\omega C_0}\right)^2\right] \left[4R^2 + \left(\frac{1}{\omega C_0}\right)^2\right]}} \quad (26)$$

For small values of C_1/C_0 all terms other than the first can be ignored. Then the alternating voltage on R is

$$u = iR = \frac{C_1}{C_0} \frac{ER}{\sqrt{R^2 + \left(\frac{1}{\omega C_0}\right)^2}} \sin \omega t \quad (27)$$

and its effective value

$$U = \frac{E}{\sqrt{2}} \frac{C_1}{C_0} \frac{R}{\sqrt{R^2 + \left(\frac{1}{\omega C_0}\right)^2}} \quad (28)$$

This voltage is independent of the frequency ω for great ω and approaches the constant limiting value

$$\frac{E}{\sqrt{2}} \frac{C_1}{C_0}.$$

Thus by keeping

$$R > \frac{1}{\omega C_0} \quad (29)$$

U becomes practically independent of the frequency. In that event, the time constant of the circuit (R, C) is great compared to the period of motion, and the test circuit of figure 16 acts like the replacement circuit (fig. 17a).

With $j = \sqrt{-1}$, we therefore get

$$\frac{U}{E} = \frac{1}{1 + \frac{1}{j\omega R C_0}}; \quad \left| \frac{U}{E} \right| = \frac{R}{\sqrt{R^2 + \left(\frac{1}{\omega C_0}\right)^2}}$$

and the effective value

$$U = \frac{E}{\sqrt{2}} \frac{C_1}{C_0} \frac{R}{\sqrt{R^2 + \left(\frac{1}{\omega C_0}\right)^2}} \quad (30)$$

But the capacity C_0' of the control arrangement and of the tube which contributes nothing toward the measuring effect is still parallel to R , hence

$$U = \frac{E}{\sqrt{2}} \frac{C_1}{C_0} \frac{1}{1 + \left(\frac{C_0'}{C_0}\right)} \quad (31)$$

according to the substitute (fig. 17b)

For $\frac{C_1}{C_0} \approx \frac{\Delta x}{d}$ it then becomes

$$U = \frac{E}{\sqrt{2}} \frac{\Delta x}{d} \frac{1}{1 + \left(\frac{C_0'}{C_0}\right)} \quad (32)$$

Design of the measuring setup. - The valuation of the design of the measuring setup is to be based on a suitable quartz pickup in which the elasticity of the pre-tension device is mild in contrast with the elasticity of a quartz column consisting of two quartzes. Subjecting such a column to a voltage of about 200 volts causes it to change its length by

$$\Delta x = 2d_{11} U$$

$$= 2 \times 2.12 \times 10^{-10} \text{ cm/V} \times 200 \text{ V} = 0.848 \times 10^{-6} \text{ mm}$$

This length change is independent of the column dimensions. Selecting the electrodes as circular disks with a diameter of 7.6 millimeters and a distance of 0.003 centimeters we get

$$C_0 = \frac{\epsilon F}{d} = \frac{0.089 \frac{\text{pF}}{\text{cm}} \cdot 0.453 \text{ cm}^2}{0.003 \text{ cm}} = 13.4 \text{ pF}$$

At a condenser voltage of 300 volts and an estimated control and valve capacitance of $C_0' \approx 20 \text{ pF}$ we get

$$U = \frac{E}{\sqrt{2}} \frac{\Delta x}{d} \frac{1}{1 + \frac{C_0'}{C_0}} = 2.25 \times 10^{-3} \text{ V}_{\text{eff.}}$$

In order to obtain a reading approximately independent of the frequency at above 1000 Hz it is necessary that

$$R \geq \frac{1}{\omega(C_0 + C_0')} = 4.77 \text{ M}\Omega$$

We decided on $R = 10 \text{ M}\Omega$, with a grid resistance of $20 \text{ M}\Omega$ across a coupling condenser of 1000 pF connected in parallel, thus making the effective resistance about $6.7 \text{ M}\Omega$.

Counting with a 100-fold excess departure in the case of resonance leaves a voltage of 0.23 volt. for the input grid of the amplitude recorder. Since this excess at resonance was not known for the time being, a measuring range from 1:1000 was provided. The absolute amount of input voltage increases with the condenser direct voltage, the upper limit of which is determined by the flash-over voltage. The input voltage is further amplified and rectified in the vacuum tube amplifier (fig. 18) so that the plate current of the output tube is an indication of the input alternating-current voltage. For accurate coverage of the 1:1000 range with the reading instrument, the amplifier is fitted with variable μ tubes and automatic control.

The input stage has a T 113 electrometer tube suitable for high grid resistance. The plate resistance of all stages is $20,000 \Omega$ in view of the amplification of the frequency band from 10^3 to 10^5 Hz , affording a sufficiently small amplification drop up to frequencies of 10^5 Hz . The grid of the second stage (AF3) is affected by the regulating voltage of a duodiode AB 2 which, at the same time, takes care of the detection of the amplified measuring voltage. The direct voltage occurring on the second plate or load resistor of the duodiode is once more amplified by a direct current amplifier stage after being filtered by a R,C member and the plate current of the last stage measured with a 10Ω (Siemens-Halske) instrument.

In this test method the pickup itself forms one electrode of the condenser microphone, thus removing the disturbing frequency limitation of the test range to 10^4 Hz and the disagreeable effect of the air path between pickup and microphone.

Provisions had been made to keep the electrodes a and b (fig. 15) under vacuum so as to raise the susceptibility of the test setup by a maximum condenser direct-current voltage. But it was found that the measuring effect below the minimum voltage of Paschen's law (air about 330 volts) was amply sufficient. Thus the disagreeable vacuum was avoided. The pickup was arranged upright. The electrode a forms the front of an elongated cylinder, which made it possible to bring it near to the bottom surfaces of pressure pickups having a gas passage as pressure transmission link. This cylinder is mounted in the center of a heavy steel plate (fig. 19) originally intended as a pump plate for a vacuum bell and keeps vibrations away from the electrode. Then the entire unit was mounted on a shock-proof table, thus assuming good body acoustic isolation. The steel plate rests on a U-iron frame, below which the amplitude recording instrument is arranged so that it can be connected to the measuring finger by means of a low capacity plug connection, with the result that the additional control capacity lowering the measuring effect, can be kept small. The pickup together with the bottom surface is advanced toward the measuring finger by means of a support with vertical micrometer adjustment, the driving spindle of which permits a reading to about a 1/100-millimeter setting. The support is mounted on an upright steel plate held by an angle-iron frame. The pickup can be mounted into the support by itself or in conjunction with a large mass. Since the movement of the masses through the spindle of the support did not appear to be satisfactory, the arrangement indicated in figure 20 was resorted to. The weight is divided over the three spindles of the locking ring. This assures correct adjustment of the capacity C_0 even with large masses. Coupling between test specimen and vibration susceptible parts of the setup are avoided by imbedding the test specimen in soft rubber. In spite of the measures to guard against body acoustics, the setup was susceptible to atmospheric sounds so that the measurements had to be made at special hours of the day.

The current source for the variable frequency was a home-made beat buzzer with a frequency up to 12 kHz, which was subsequently replaced by a Siemens-Halske product having a 100 Hz to 100 kHz frequency range. The frequency uncertainty after calibration amounts to $0.005 f \pm 25$ Hz. However, since this buzzer supplied voltages of only about 20 volts, it had to be raised to about 150 volts by a plate fed last stage. The complete setup is shown in figure 21.

V. MEASUREMENTS

The measurements were made with the following ends in view:

(1) The piezoelectric pickups were to be explored in the range between 1 and 100 kHz.

(2) For the recording of the resonance curves, only relative values of the ordinates are required.

(3) The reading of the amplitude recorder in the neighborhood of 1 kHz is very little dependent upon the frequency and the amount of capacity C_0 . The latter is to be kept as constant as possible.

(4) Because of the automatic control of the variable μ tubes, the reading is approximately logarithmic rather than linear.

(5) The amplitude recorder can be calibrated with the hook-up shown in figure 17b; it affords a voltage which at frequency f produces a deflection $\Delta\alpha$ of the reading instrument and serves as a relative indication of the motion amplitude of the pickup bottom.

1. Frequency Variation and Calibration

Curves of Amplitude Recorder

Following a number of preliminary tests for perfecting the equipment, the frequency variation of the amplitude recorder, shown in figure 23, was determined on the hook-up indicated in figure 22. It is a trifle higher at higher amplitudes than at lower amplitudes. This is due to the fact that, because of the automatic control of the instrument, the operating point of the variable μ tube shifts at higher amplitudes toward the lower, more curved part of the tube characteristics so that the higher harmonic volume increases because of greater distortion. But the higher harmonics are not evenly amplified so that the amplitude is weakened.

The findings for three frequencies are shown in figure 24. These curves are valid for a definite capacity

C_0 only. Consequently the correct C_0 had to be established every time during the pressure pickup investigations. It was accomplished by adjustment of a corresponding distance of the bottom surface of pickup b and the opposite electrode a, as the internal capacity of the amplitude recorder remains the same.

2. Study of Piezoelectric Pickup of Various Types and Sources

The study included the following types:

- (1) Piezoelectric pickup, manufacture A (fig. 12)
- (2) Piezoelectric pickup, manufacture B
- (3) Piezoelectric pickup, Meurer type (fig. 6)
- (4) Piezoelectric pickup with prestressed flat spring (fig. 26)
- (5) Special design with welded spring bushing (fig. 27)
- (6) Special design with threaded spring bushing and with gas seal (fig. 28)
- (7) Same as 6; ready to operate, with gas seal
- (8) Spring bushing as in 5 and 6 (fig. 29)

Some of the pickups were tested with and without a threaded mass substituting for the engine casing mass. Since the resonance curves below 1000 Hz were not determined, the subsequent curves carry the ratio x/x_a as ordinate, x denoting the deflection of the pickup bottom at frequency f and x_a the deflection at the beginning of the resonance curve, usually at 2000 Hz.

In order to emphasize the resonance curves only the measured points shown as small rings are given in figure 31. The test points are just as close in all curves. The resonance curves were slowly and evenly covered so as to definitely secure all maximum and minimum values with closely spaced test points. Duplications of the measurements with the same setup showed complete agreement of the curves.

Piezoelectric pickup, design A (wt. 161.8 grams; fig. 12; resonance curve, fig. 30).-- This pickup is characterized by the mechanical prestress of the quartz column accruing from a comparatively heavy flat spring made of one piece with the lower part of the casing. This makes the pre-tension dependent upon the temperature of the flat spring and the tightening of the threaded connection due to the threads of the pickup in the spark-plug hole. The pressure transmission link is a piston. No water cooling was provided.

Curve a, piezoelectric pickup without mass: The first minor departure from the initial deflection occurs at 3750 Hz, followed by another at 4750 Hz. There are two fairly undamped maximum values at 8950 and 9500 Hz, which should be regarded as natural frequencies. The maximum at 9500 Hz might be attributable to the mass of the pressure transmission piston, elastically supported on the quartz column and the elements in between.

Curve b, piezoelectric pickup with mass: On loading the pickup with a mass (fig. 20) of about 3.6-kilogram weight* the maximum values shift considerably and new maximum values occur. Herewith the practicability of the pickup is restricted to the range up to about 2000 Hz. Incidentally it is pointed out that the position of the maximum values can be varied only through the varyingly severe tightening of the pressure pickup while attaching or removing the mass in the different test series. This is especially true on this pickup design because the pre-tension of the quartz column is produced by a flat spring which is machined from one piece with the casing.

Piezoelectric pickup, design B (wt. 183.9 grams; resonance curve, fig. 32).-- The quartz column of the piezoelectric pickup comprises eight quartz plates which receive their mechanical initial stress largely from a flat spring, which, however, is not solidly built in on the edge, but whose edge rests only on one side while the center of the opposite side presses against the quartz column. The link transmitting the pressure is an elongated piston secured by a thin diaphragm which presses it against the flat spring and seals the pickup against gas. There is no water cooling.

*Hereinafter the weight is always to be construed as the weight of the loading masses.

Pickup with mass of about 3.6 kilograms: The first minor departure from the initial deflection showed at about 5500 Hz, the first major departure at 8200 Hz. The natural frequencies are located at about 16,600 and 18,000 Hz. It is practical for recording vibrations up to about 5000 Hz. The maximum values are damped very little.

Piezoelectric pickup according to Meurer (wt. 268.0 grams; fig. 6; resonance curves, fig. 33).— This pickup has water cooling. The mechanical prestressing of the quartz column is effected by spring bushing. Its shortcoming lies in the gas passage as pressure transmission link.

Pressure pickup with mass of about 3.6 kilograms: The first departure from the initial deflection occurs in curve a at 6500 Hz. The maximum values are situated at 23,600 and 24,400 Hz. Since the departures at 11,900 and 12,300 Hz appeared to be due to higher harmonic waves of the exciting voltage, a second test series (curve) was carried out with the mass before the quartz plates increased by a threaded additional mass. This shifted the maximum values from 23,600 and 24,400 to 15,050 and 16,300 Hz, but the departures at 12,000 Hz and that at 6500 Hz remained at the old place, which did not support the previous suspicion, and at the same time proved that the maximum values at 23,600 and 24,400 Hz are actually attributable to the oscillation of the mass before the quartz. This pickup would be practical up to about 6000 Hz if the pressure transmitting passage did not already cause difficulties at low frequencies.

Piezoelectric pickup with pre-tension flat spring (wt. 245.3 grams; fig. 26; resonance curve, fig. 34).— This pickup also has a water-cooling system consisting essentially of an annular channel in the bottom of the casing. The mechanical pre-tension of the quartz column is accomplished by a flat spring of about 100 kilograms.

Pressure pickup with mass of about 3.6 kilograms: The first departure from the initial deflection occurs at 3050 Hz, the maximum resonance values at 6250 and 8750 Hz. This pickup can therefore be used for recording vibrations up to about 2000 Hz. Since this, like the pickup of design A, shows maximum resonance values at very low frequencies, it is concluded that such pickups with pre-tension flat springs are not suitable for high-frequency measurements.

Pickup of special design with welded spring bushing but without gas sealing membrane (wt. 66.5 grams; fig. 27; resonance curves, fig. 35).— The outstanding feature of this design is its small bulk, notwithstanding its satisfactory water cooling. The plug is spot-welded into the spring bushing which furnishes the mechanical pre-tension of the quartz column. The spring extends into the threaded connection and provides a smooth fit of the pickup bottom, hence eliminates separate pressure transmission links. The pin of the middle electrode is positively connected with the plug pin by means of a very flexible spiral spring which amply insures mechanical uncoupling. The gas seal necessary in pressure measurements was omitted in this test.

Pressure pickup with mass of about 3.6 kilograms: The first departure in curve a occurred at 11,300 Hz and was trifling. Only one resonance maximum occurred at 44,300 Hz and it was weakly damped. To denote the effect of the mass of the plug on which the amplifier cable is connected, the aluminum-plug bushing was replaced by a 30.7-gram mass of steel. The resonance curve maintains its character, as indicated by curve b.

The advantage of this over the first four other pickups is startling. It can be used up to about 11,000 Hz as oscillation recorder.

Pickup of special design with threaded spring bushing without gas sealing membrane (wt. 64.2 grams; fig. 28; resonance curves, figs 36 and 37).— The threaded spring permits removal of the quartz from the bushing for cleaning. The gas sealing membrane was not in operation during the recording of the resonance curves in order to preserve comparability with the preceding curves.

Pickup without mass: This pickup was first explored without mass loading (fig 36, curve b). The first departure from the initial deflection occurred at the very low frequencies of 1000 to 3000 Hz. The cause of it was traced to the connecting rod between central quartz electrode and pin plug which starts oscillations in the pin and thus causes the departure. After the originally rigid connection of the central quartz electrode with the pin plug had been replaced by a very flexible, thin spiral spring, the cause disappeared (fig. 36, curve a). However, the next test series with the improved pickup installation disclosed that the minor departure occurring

at about 12,300 Hz had shifted toward 10,900 Hz. This proves that the location of the resonance is also dependent upon the contingencies of the restraint in the assembly so that pressure pickups of the same design may manifest somewhat dissimilar resonance curves.

Piezoelectric pickup with 3.6- and 0.73-kilogram mass. A change in mass from 3.6 kilograms (fig. 37, curve a) to 0.73 kilogram (curve b) produced no marked changes in the lower frequencies, nor could a 30.7-gram increase in contact mass be detected on the resonance curve (curve c). In contrast to pickup 5 the principal resonance area is situated at about 40,500 Hz, or 3800 Hz below that of the welded version. In accord with this the welded connection is superior to the threaded connection for achieving higher natural frequencies for equal spring pre-tension.

Pickup of special design with threaded spring bushing, finished with gas sealing membrane (wt. 68.5 grams; fig. 28; resonance curves, fig. 38).— The gas sealing membrane required for gas pressure recording is 0.05 millimeter thick. It is attached on the bottom of the spring bushing with a screw of heat-resistant steel, which forms the bottom closure of the pickup. A threaded ring of the same kind of steel holds the membrane on the casing, the gas seal consisting of thin copper gaskets. The clearance between the screw and the threaded ring is 0.2 millimeter so the forces on the membrane are small and prolong its life. The bottom of the pressure pickup is smooth except for the 0.2 millimeter wide and 1.2-millimeter deep ring slit. The cooling of the casing and of the spring bushing in the threaded connection has proved satisfactory even by detonating operation.

Pickup with 3.6-kilogram mass: The resonance curve manifests only minor variations. The first departure from the initial deflection occurs at 11,300 Hz, the principal resonance at 39,000 Hz. With knowledge of the frequency to be measured, this pickup would insure amplitude records practically up to 30,000 Hz with relatively small errors.

In the tests, so far, the loading mass had been screwed to the pickup over an iron-asbestos ring exactly as for engine operation. This ring was now replaced by an aluminum ring (curve b). It resulted in a new departure at about 4500 Hz, which probably corresponds to the oscillation of the sound concentration formed by pickup mass, loading mass, and elasticity of screw joint. This oscillation is not detectable when the iron-asbestos ring is used.

A series of tests was also made on this pickup with increased mass before the quartz (curve c). The principal resonance areas were found to have shifted downward. The maximum departure occurred at 33,200 Hz. In addition, new resonance areas occurred. The entire curve becomes more uneven in the low frequencies. The first minor departure, however, remains at its position of 11,300 Hz.

Springs of the pickup of a special design with welded or threaded thrust block (fig. 29; resonance curves, fig. 39).— The two springs differ only in the closing piece. While the welded version is simple to manufacture the threaded bushing requires much more care, especially during assembly because of the linear dimensions involved.

In order to prevent corrosion through the cooling water, the bushings should preferably be of rustproof steel or else rustproofed. Since the bearing surfaces of the quartz crystals require a carefully worked and level base, the lowest electrode is formed by a steel disk machined on one side which is pressed in the bushing bottom between an aluminum gasket which causes the aluminum to spread and fill the space in between. The resonance curves indicate the superiority of the weld. The welded version (curve a) with a weight of 9.1 grams as against 15 grams for the threaded (curve b) indicates a very favorable course of the resonance curve as far as the principal resonance area; the sole minor departure occurs at 35,100 Hz. Then there is no more departure up to the principal resonance which is located at 70,000 Hz. It seems as if the departure at 35,100 Hz were due to a higher harmonic of the exciter voltage. There are smaller maximum values at 73,000 and 73,500 Hz and ultimately a slight departure at 77,600 Hz. In contrast hereto, the threaded version (curve b) manifests the first departure at 22,000 Hz. The first major departure occurs at 48,400 Hz, the principal resonance at 62,400 Hz. On the whole, the curve is much more uneven than for the welded version.

VI. FINDINGS

The resonance curves of the piezoelectric pickups indicate that even the best design types explored here manifest departures from the initial value even in the approximately linear portion, and they are far greater than the errors involved in static measurements. Hence, static

calibrations of piezoelectric recorders maintaining an accuracy of 1 percent or better can be fully utilized only if, in addition to the knowledge of the resonance curve, the frequency of the vibration under investigation can be determined. The experiments indicate the superiority of pickups 5 and 7 over the others. Conclusions can be drawn concerning the design of such pressure pickups and the frequency ranges obtainable.

To begin with, the design should be as simple as possible so as to afford oscillation patterns with few degrees of freedom; the more complicated the design the more degrees of freedom must be expected.

The pre-tension by spring bushing is superior to that by flat spring.

The mass before the quartz should be as small as possible, even though the frequency scope is not solely governed by the mass before the quartz, but by much lower natural frequencies obviously due to the oscillations of the casing or the threaded connection. Even so, the action of the masses before and behind the quartz is contributory toward the distortion of the resonance curve of the system (m_2, c_2). Admittedly the ratio of the masses could be kept at a minimum by increasing the mass behind the quartz although mass concentrations behind the quartz conceal the hazard of formation of new configurations susceptible to oscillation.

Screw connections should be kept at a minimum. The welded joint seems superior to the threaded joint.

Gas passages with or without damping should be avoided, since they frequently shift the upper frequency limit much below that obtainable on the mechanical oscillation system. The amplitudes of pressure vibrations can be obtained with acceptable accuracy up to 10,000 or 11,000 Hz with pickups 5, 6, and 7, and frequencies as high as 26,000 Hz by admittance of greater errors within a limited frequency range.

If the requirement for the measurement does not specify the use of the spark-plug hole, the welded spring bushing could, for instance, be mounted direct into the wall of a combustion bomb. In that event, frequencies of 50,000 to 60,000 Hz are recordable. Since the explored spring bushings were designed for installation in pressure

pickups of engine cylinders, they are readily adaptable to combustion bomb use by making the bushing bottom thinner and using foil for the middle electrode of the quartz. This should raise the principal natural frequency to 100,000 Hz.

The employed test method has proved satisfactory, and so closes the gap heretofore existing in the correct analysis of piezoelectric recorders. The study of pressure pickup, recorder, and accessories now permit a satisfactory appraisal of the oscillation characteristics of the whole test arrangement.

DISTURBANCE OF PIEZOELECTRIC RECORDER

READINGS THROUGH MASS FORCES

In the pressure measurements on engine cylinders by piezoelectric recorders the gas forces are superposed by disturbing mass forces produced by the pickup. Methods of combatting these forces are described. The elastic suspension of a measuring quartz is analyzed; heretofore unobserved potential errors were appraised and checked by experiment. Suggestions for removing the sources of errors are outlined.

VII. PROBLEMS

The aim in engine research to measure pressure processes with a minimum of inertia has brought about the development of the piezoelectric recording method which affords the correct measurement of much more rapid pressure changes than any other known method.

Its practical limitation is governed by the oscillating properties of the piezoelectric pickup. Unfortunately this method is affected with mass forces which, in the piezoelectric pickup illustrated in figure 4Q, are substantially due to the mass of the pressure transmission links d and b and to a lesser extent to the mass of quartz a. Eliminating the pressure transmitting piston d, replacing the central electrode b by a thin foil, and using smaller quartz plates results in a marked

reduction of the mass forces. At accelerations equal to 1000 times gravitational acceleration, for instance, a quartz cylinder of 6 millimeters in diameter and 3 millimeters in height would produce a mass force of about 0.220 kilogram. Since forces acting on the quartz in engine pressure measurements are usually by about two orders of magnitude higher, the problem of mass forces would be largely solved if the mass of the pressure transmission links could be eliminated. It would even afford high natural oscillation characteristics for the pickup, since the mass forces and oscillation characteristics are jointly determined by the masses existent in the pickup. Because of the thermal stress on the pickup in the engine cylinder some mass, even though small, before the quartz is unavoidable. As a result different ways of making the mass forces ineffective for the reading have been attempted.

VIII. CONVENTIONAL ARRANGEMENTS

WITH LOWER MASS FORCE READING

1. Use of Double System

The mass force reading can be minimized, according to Postlethwaite (reference 26), by the use of two systems in the pickup unit: a pressure system and a vibration system. But the second system responds only to the accelerations, not to the gas pressure. The vibration system is arranged to deliver an equal and opposite electrical charge from that delivered by the pressure crystals. Unfortunately, this measure results in making the pickup fairly big and its cooling difficult and probably renders the exact balance of the systems difficult. In addition, the second system creates a new vibration pattern.

2. Use of Balanced Piezo with Different

Piezoelectric Modulus

Bekesy (reference 22) utilizes the relation of the piezoelectric constant and the direction of cut of the quartz. By appropriate choice of modulus and of the masses of the system, a vibration unit unresponsive in pickup axis direction can be built up with two quartz crystals. According to figure 41, for instance, the

quartz masses are identical. The piezoelectric modulus of the upper quartz is only a third of that of the lower, both pieces of quartz being arranged to deliver opposite electrical charges. Thus, the mass of the central electrode is twice as great as that of the cover plate. By reason of the differential connection of the two quartz crystals, the sensitivity drops to one-third of that of the conventional quartz arrangement, for which the insensitivity is exchanged for reduced vibrations.

3. Elastic Suspension of Quartz

Lastly, there are the attempts by Fahrentholz, Kluge, and Linckh (reference 23) to minimize or suppress the effect of the mass forces above a specified frequency by elastic suspension of the quartz. In the diagrammatic sketch of the pickup (fig. 42) one quartz is suspended between two flat springs which maintain the quartz simultaneously under initial tension, the upper one serving as tapping electrode for the quartz charge and being therefore highly insulated. The quartz surfaces are curved.

Because of its simplicity, this arrangement is studied in detail.

IX. THEORETICAL ANALYSIS OF THE ELASTIC SUSPENSION OF QUARTZ BETWEEN TWO FLAT SPRINGS

Notation

A	proportional factor (V/kg)
a	motion amplitude (cm)
b	acceleration (cm/s ²)
c	elasticity (kg/cm)
F	surface (cm ²)
f	frequency (Hz)
f _{1,4}	natural oscillation characteristics (Hz)

m	mass ($\text{kg s}^2/\text{cm}$)
P	force (kg)
p	pressure (kg/cm^2)
x	deflection (cm)
x_a	deflection of mass m_2 (cm)
ω	angular frequency = $2\pi f$ ($1/\text{s}$)

1. Natural Vibration Characteristics and Mass Forces

What is the conduct of this system relative to its natural vibration characteristics, the mass-force indication, and the gas-pressure reading? As to the first, the resonance curve of the system is largely informative. In order that the static calibration retain its validity at high frequencies as well, the resonance curve must manifest no marked departures throughout the entire frequency range concerned. Resonance curve and static-calibration curve should retain validity even for simultaneous appearance of gas pressure and mass forces. Information concerning the resonance curve can be obtained by first attempting to establish the natural vibration characteristics of the system. The system indicated in figure 42 can vibrate preferably in two natural vibration modes. In the first mode the flat springs lying against the quartz execute opposite motions; in the second, quartz and flat springs vibrate synonymously against the casing. The natural vibration characteristics f_1 of the first which is relatively high depends upon the spring mass, the size of the quartz, and the properties of the contact surfaces. Since the masses of the flat springs load the quartz, f_1 is situated correspondingly lower than the natural vibration characteristics of the quartz alone. Several hundred thousand vibrations per second should be attainable.

The vibration characteristic f_2 of the second mode, indicated in figure 43 by the mechanical substitute pattern of this mode, is defined in the sum m_2 of quartz masses m_Q and flat spring masses m_F and, in addition, by the elasticity c_1 and c_2 of both springs relative to the casing mass m_C , which may be assumed to be very great. It is to be noted that c_1 and c_2 represent the

concentrated forces that effectuate a deflection of 1 centimeter. It is expedient to visualize these deflections as being so small at first that the elasticity values may be considered constant. By major deflections, whether due to mass forces or gas forces or both, this is, of course, no longer permissible. The f_2 of two flat springs 0.2 millimeter in thickness and 8 millimeters in diameter and a 3- by 6-millimeter quartz amounts to several thousand vibrations per second. More flexible springs would put f_2 lower but, at the same time, would also lower the pressure-recording range for reasons of strength.

The effect of vibrations of the piezoelectric pickup on the tension at the quartz, that is, on the reading of the instrument, is analyzed first.

If the casing mass m_2 of the substitute vibration system indicated in figure 43 executes vibrations under the effect of acceleration alternations of amplitude b , of frequency f and amplitude a , the deflection x_a of mass m_2 , with damping discounted, is:

$$m_2 \frac{d^2 x_a}{dt^2} + c(x_a - a \sin \omega t) = 0 \quad (33)$$

After the transitory state a simple calculation gives the amplitude of spring travel $x = x_a - a$ of spring c with $c/m_2 = \omega_2^2$ at

$$x = - \frac{b}{\omega_2^2 - \omega^2} \quad (34)$$

and the mass force amplitude of mass m_2 at

$$F_2 = m_2 \omega^2 (x + a) = cx = - m_2 b \frac{\omega_2^2}{\omega_2^2 - \omega^2} \quad (35)$$

If the static calibration is to be used for the vibration measurements, the operating frequency of the piezoelectric pickup of figure 40 must, because of the

departure indicated in figure 44, remain small compared to the lowest natural vibration characteristic of the resonance curve. Accordingly, as mentioned above, the lowest natural vibration characteristic is preferably shifted upward as much as possible. For that very reason, the occurring acceleration frequencies themselves will ordinarily be small compared to the lowest natural vibration characteristic. In this event the mass force in the quartz is, according to equation (35), without allowance for the masses of the piston and the electrodes:

$$P_Q = - m_Q b \quad (36)$$

But, if the natural vibration frequency of the second type of vibration system of figure 42 can be made small relative to the frequency f of the exciting acceleration, as obtainable by flexible mounting, we get

$$P_Q = \left(\frac{f_2}{f} \right)^2 m_Q b \quad (37)$$

that is, the mass force in the quartz is reduced in the ratio $(f_2/f)^2$. Furthermore, appropriate choice of size of flat springs makes it possible to keep the mass force of the quartz, which itself is small compared to the gas forces, piezoelectrically ineffective (reference 23). The potential difference U_Q produced by it at the small deflection x is proportional to the amount $x(c_2 - c_1)$. According to equation (35) we get

$$x = \frac{P_2}{c} = \frac{P_Q}{c_1 + c_2}$$

with $c = c_1 + c_2$, hence the difference in potential

$$U_Q = A \frac{1 - \left(\frac{c_1}{c_2} \right)}{1 + \left(\frac{c_1}{c_2} \right)} P_Q = A \alpha_m P_Q \quad (38)$$

where for abbreviation
$$\frac{1 - \left(\frac{c_1}{c_2}\right)}{1 + \left(\frac{c_1}{c_2}\right)} = \alpha_m, \text{ and } A \text{ indicates}$$

a constant denoting the relation between potential and force. U_Q becomes zero when $\alpha_m = 0$, that is, $c_1 = c_2$. In that event the mass force of the quartz is not included in the measurement. In relation with the frequency f of the exciting casing vibration the results are:

Within range $f < f_2$ the mass force of the quartz is fully effective.

Within range $f > f_2$ it is reduced in the ratio

$$\left(\frac{f_2}{f}\right)^2$$

Within range $f \approx f_2$ it increases correspondingly to the departure at resonance of the system.

In all cases including that of resonance f_2 it will not be indicated if $c_1 = c_2$.

The next problem is to establish the extent to which the condition $c_1 = c_2$ or $c_1/c_2 = 1$ can be satisfied by an arrangement as indicated in figure 42. The vigorous mathematical solution of the spring characteristics seems scarcely feasible, since the calculation of a flat spring rigidly clamped on the edge and loaded on one side under gas pressure, on the other by a concentrated force, is unknown so far. But in order to gain some insight into the possible aspect of similar spring characteristics, some simplifying assumptions - to be checked experimentally as to their reliability - are made.

It is presumed that both flat springs have identical spring characteristics, as heretofore always tacitly assumed in the literature. It is further assumed that the two springs are identically stressed by uniformly distributed load, since this case of the clamped circular plate with great deflection is adequately covered by empirically proved approximate formulas (reference 25).

The example serving as basis of the present appraisal relates to two flat springs of 0.2 millimeter in thickness and 8 millimeters in free diameter between which a 3-millimeter-thick quartz with curved surfaces is clamped under 9 kilograms of initial tension. The pressure relation with the contact surfaces created by the curvature of the surfaces is not considered.

Figure 45 shows the characteristics of the two flat springs thus computed, x being the deflection of the quartz from rest position. Since the quartz must be maintained under initial tension, the flat springs are bent along different directions in the undeflected state of the system.

This explains why the lower spring under load from below can become more flexible, while the upper becomes definitely stiffer. The spring characteristics of figure 45 afford the elasticity values of figure 46, which agree with reality as will be shown later. Herefrom it may be concluded that the reading of the mass force is not suppressed at great deflections, because $\alpha_m = 0$ only by identically great elasticities.

2. Gas Pressures

Strictly speaking, the external force produced by the pressure p is evenly distributed over the entire lower flat spring. A certain portion is absorbed by the restraint, while a much smaller portion is transmitted to the upper flat spring by the quartz located behind it. Since this portion acts as a concentrated force on the upper flat spring, the present loading condition of the upper spring is the same as previously for the mass force. The transmitted and hence indicated portion of the force is

$$P_a = c_2 x \quad (39)$$

If the load on the lower flat spring were a very small concentrated load rather than the uniformly distributed load, about half of it would be taken up by each of the two identical springs. This had been the general assumption in the theoretical treatment of the elastic suspension of a quartz under any gas pressure (reference 24). But in reality, the portion taken up by the second spring is much less than half the product of the force from spring

surface F_f and pressure p . Consequently it is possible to assign a utilization factor α_p expressed by

$$\alpha_p = \frac{P_a}{F_{fp}}.$$

According to equation (39), the portion P_a of the force is not affected by the nonlinearity of the spring characteristic, hence the static pickup characteristic itself is nonlinear at greater deflections.

X. EXPERIMENTS

As a check on these arguments, experiments were made for the purpose of defining the spring characteristics and the relative elasticity values of a system conformable to figure 42. In this experimental setup the quartz element was replaced by an identical lens-shaped steel element. The experiments with uniformly distributed load were made by means of oil pressure and recorded by a manometer calibrated by weight loading on a pressure balance. The deflections were obtained on a dial micrometer with an indication of 1/100 millimeter per scale division. The concentrated load was applied over the lens-shaped steel element of a 15-millimeter curvature radius by balance pan and weights to avoid frictional errors. The setup enabled the recording of the spring characteristics of the entire system under hydrostatic pressure, the characteristic of the lower flat spring under combined hydrostatic pressure and concentrated load, and the characteristic of the upper spring under concentrated load. And from these data the quantities necessary for appraising the system can be obtained. The particular system consisted of two spring-steel plates of 0.213 millimeter in thickness and 8 millimeters in free diameter and the lens-shaped steel element with curved surfaces of 6 millimeters in diameter and 2.978 millimeters in thickness. The tests were made with two different initial tensions, both of which were substantially below that assumed in the appraisal for reasons of clarity. The initial tensions were 2.15 and 5.40 kilograms obtained by shims of different thickness placed between the flat springs of 2.902 and 2.800 millimeters in thickness, respectively. Shims and clamping rings consisted of smoothly ground steel washers of 15.5 millimeters outside diameter like

the spring steel plates. The spring characteristics were plotted from averages at increasing and decreasing load, which usually varied by less than 3 percent and were readily reproducible. It is therefore not very likely that the yield point had been reached. The various experimental equipment is illustrated in figures 47, 48, and 49.

Figure 50 shows the recorded deflections x of the total system for the two initial tensions; the stiffness is, of course, greater by greater initial tension as may be expected.

Figure 51 (solid curves) indicates the deflections x of the bottom spring under combined hydrostatic pressure p on one side, and concentrated load P on the other side, with p as parameter. The lines of higher pressure manifest opposite curvature to that of the lower pressure. The dot-and-dash curve indicates the deflections x of the top spring F_2 under concentrated load.

From these curves the desired quantities: elasticity values c_1 and c_2 , the force P_a acting on the quartz, and the consequent values α_m and α_p in relation to the deflection or the outside pressure can be obtained, as exemplified in figures 52, 53, and 54; the deflection force P_a of the top spring being computed with allowance for the known initial stress.

XI. RESULTS AND CONCLUSIONS

The experiments indicated that the requirement $c_1/c_2 = 1$ so important for the balance of the mass forces, is only approximately fulfilled within a very restricted range. The curve of the elasticity data (fig. 52) is somewhat different from that of the estimation (fig. 46), which is evidently because of the simplifying assumptions. The numerical data are markedly different, but the ratio c_1/c_2 is little affected by it. While the elasticity data are identical for all deflections at zero pre-tension according to the estimation, the test showed that it is actually not permissible even without pre-tension to make calculations under the assumption that $c_1 = c_2$ in the deflected state unless special measures are taken. Both

springs would have to be released from the initial tension and preferably be subjected to concentrated load. In that event it can be expected that the mass force is not indicated at any deflection. On differently loaded flat springs α_m is zero only in the rest position, but mass forces are indicated by deflections. In the model problem at vibrations equal to 1000 times gravitational accelerations, as may happen on a detonating engine, the mass-force reading due to quartz mass at 40 atmospheric gas pressure and 2.15-kilogram pre-tension will, according to the test data, amount to about 0.022 kilogram below f_2 , since $\alpha_m = 10$ percent according to figure 54. In contrast, the reading due to gas pressure is 2.8 kilograms (fig. 53), that is, at an exceptionally high acceleration the mass-force reading amounts to less than 0.8 percent, and at lower pressures it is even less. Consequently the mass-force reading by the described arrangement is small even with incomplete balancing action. Not until other masses, such as that of a pressure transmission link, become additive does the interference increase. This particular case is being dealt with elsewhere.

From the magnitude of the pressure-force factor α_p (fig. 54) it is apparent that the partial force P_a applied on the quartz is considerably less than half of the total pressure force. It does not remain proportionate to the pressure at greater deflections. This together with the relation of c_1/c_2 to the deflection causes the static characteristics of the measuring system to become nonlinear in our example, as reflected on the thin straight lines of figure 53. In that manner, higher harmonics are induced in the output distribution by sinusoidal pressure alternations. According to the experimental data the output control at the smaller pre-tension should therefore not be permitted to take place at much over 20 atmospheres.

If the pressure acting on the measuring system with natural frequency f_2 is, instead of constant, a periodically variable pressure force $P_p = P_{p0} \sin \omega t$ with constant amplitude at variable frequency f_1 , the deflection x of the quartz from rest position follows a resonance curve conformable to figure 55 which, with damping and nonlinear effects discounted, follows from

$$m_2 \frac{d^2 x}{dt^2} + c x = P_{po} \sin \omega t \quad (40)$$

whence we get after the transitory state, with $\frac{c}{m_2} = \omega_2^2$:

$$x = x_0 \frac{1}{1 - \left(\frac{\omega}{\omega_2}\right)^2} \quad (41)$$

where x_0 is the static deflection.

In the resonant zone $f \approx f_2$ the nonlinear properties of the spring characteristics are especially effective by reason of the anticipated very great deflections attending the resonance departure by factor ρ . In this instance, the mass-force reading would be especially pronounced and the test data would be accompanied by frequencies not contained in the instrumental quantity. In order, therefore, to minimize the nonlinear effects in the zone of resonance, the measuring range outside of the resonance should be restricted to about the ρ th part of the approximately linear section of the characteristics, since in most alternating pressure measurements, for instance, on engines, a range of frequencies which may contain the relatively low frequency f_2 must be considered. The result may be a very restricted pressure-recording zone free from distortion. The nonlinear characteristics are especially unwanted in gas-pressure measurements because the gas pressure is usually superimposed by a constant pressure portion which shifts the operating points even further into the nonlinear zone.

According to figure 55, the deflection above the resonance frequency f_2 drops quickly below the static frequency and the question is how low it can sink at all and still assume a correct reading. Above f_2 the driving force, in this case supplied by the gas pressure p , is largely utilized to accelerate the masses of the system, while below f_2 it previously strained the springs. Firstly, it is noted that above f_2 as a result of the decreasing deflections the nonlinear properties of the springs recede, as a result of which high-frequency mass forces are much better removed than low-frequency ones.

Then, in order that the reading of p above f_2 is not falsified, the masses of the flat springs must be in the same mutual ratio as that of the elasticities, previously. The reading then will be correct so long as the deflection remains great in proportion to the compression of the quartz and to the surface layers. This ceases to hold on approaching the natural vibration frequency f_1 .

So for a pickup conformable to figure 42 the following is applicable: a complete suppression of indication of the mass force is secured when the control of the output is restricted to the zone in which the elasticity values of the two springs are practically in agreement. The quantity $p \sin \omega t$ in the neighborhood of the resonance $f = f_2$ is reproduced without distortion when the deflection at frequencies outside of the resonance is restricted to the p th part of the linear section of the static-pressure pickup characteristics, because this is the only instance where, even by resonance, the linear part of the characteristic can be adhered to. Moreover, since mild flat springs are desirable for reasons of lower mass forces, the design of figure 42 is particularly suitable for low pressures. Heavier springs may enlarge the linear zone in a given case, but without changing anything in the nature of the phenomenon. Besides, f_2 would then be located undesirably high.

XII. SUPPLEMENTARY MASSES

The elastic suspension of the quartz between two flat springs presupposes the measuring system not to be subjected to high temperatures. This rules out the described arrangement for use in the cylinder wall of an engine, since the hot combustion gases would change the spring characteristics completely. It was suggested, therefore, to use a pressure-transmission piston m_k again and to equalize its mass forces by dissimilar spring stiffness, figure 56 (reference 24). But in that event the already-existing vibration possibilities would be supplemented by the longitudinal vibration of the piston with the frequency f_3 of the first harmonic.

This natural vibration frequency provides that its wave length must be comparable with the length of the piston. For steel a 30-millimeter piston length would yield f_3 at about 40 kHz. However, these frequencies are not removed by the elastic suspension but must be considered as

natural vibration frequencies of the system. Natural frequencies of this order of magnitude are obtainable with rigid thrust block as well, if care is used, as curve b in figure 44 indicates, except that with a rigid thrust block no exclusion of the mass forces is possible even at low frequencies.

In order to remove the mass force P_k of the piston mass m_k of the type indicated in figure 17 at low-vibration frequencies, it was again suggested (reference 24) to select dissimilar flat springs, preferably so that the conditions

$$c_1 x = P_k + \frac{1}{2} P_Q \quad \text{and} \quad c_2 x = \frac{1}{2} P_Q$$

are satisfied. Then

$$\frac{c_1}{c_2} = \frac{2 P_k + P_Q}{P_Q} = \frac{2 m_k + m_Q}{m_Q} \quad (42)$$

Now the bottom spring is much heavier than the one on top and the sensitivity of the pressure pickup much lower. To illustrate: on a 6-millimeter-thick piston 30-millimeters long and a quartz of 7 millimeters in diameter and 5 millimeters high, little less than 3 percent of the total pressure force is transferred to the quartz. This necessitates an exceptionally high amplification of the measuring effect in order to obtain the deflection voltage required for the control of the output of an electron-ray oscillograph. This shortcoming can be avoided, according to the same suggestion, by disposing a mass m_a equal to the piston mass m_k behind the quartz conformable to figure 57. But this produces a further natural vibration at which masses m_k and m_a vibrate oppositely, while the quartz and the not completely fitting contact surfaces form the spring link. The natural frequency f_4 of this system is fairly low since quartz and metallic surfaces, even with the highest polish, do not fit perfectly without very high mechanical initial tensions, and therefore represent relatively mild springs (reference 17). And high initial tension is not permissible, as shown elsewhere, because of the nonlinear spring characteristics,

since it shifts the operating points too far into the non-linear zone, with the result that the natural frequency of a pickup conformable to figure 57 may be lower than obtained with a rigid thrust block, where high initial tensions up to 100 kilograms or more are admissible and common. The arguments for the correct reproduction of $p \sin \omega t$ are similar to those advanced for the arrangement in figure 42.

Through the pressure transmitting piston and its equalizing mass the mass forces to which the springs are subjected become particularly great when piston, quartz, and balancing mass vibrate synonymously against the casing conformable to the natural frequency f_2 of the pressure pickup of figure 42. To illustrate: for a steel piston of 6 millimeters in diameter and 30 millimeters in length and provided with compensating mass, the weight on the flat springs is about 13.5 grams. At vibrations equal to 1000 times the gravitational acceleration the springs would be additionally loaded by 13.5 kilograms and, in addition, by the stress due to the gas pressure. If the excitation is by means of vibration frequencies at the corresponding resonance frequency f_2 , the load becomes even greater because of the rise above resonance, of which then, because of the dissimilar elasticities at great deflections, a fraction α_m is indicated. In this instance, the initial stress through the suspension springs is especially unfavorable.

XIII. SUGGESTIONS FOR AVOIDING NONLINEAR DISTURBANCES

The nonlinear disturbances of the reading by the pickup characteristic and the mass forces at great deflections are attributable to the mechanical initial tension on nonlinear flat springs and to the variation of the ratio of the pressure force portion P_a reaching the quartz to the total pressure force F_{fp} at variable pressure.

The first cause can be removed by using springs with linear characteristics and leaving the initial tension to the springs, or else resort to nonlinear springs of identical characteristics and transmit the initial tension of a special pre-tension device. The latter can be accomplished, for instance, by a tension spring conformable to

figure 58, which either passes through the quartz or, as in figure 59, on the sides of the quartz or arranged in rings about the quartz to form a tubular spring around the quartz, which also makes water-cooling possible. It might even be appropriate to use two quartz in the conventional manner with a small central electrode. This would, while lowering the natural vibration frequency, make the design of the measuring system substantially simpler. But frequencies above 100 kHz might still be unattainable, according to experiments.

The second cause is probably best avoided by keeping the bottom spring free from the uniformly distributed stress and attempting the use of a gas-sealing membrane conformable to figure 59, which leaves only a very minute clearance between the pickup casing and the rigid pressure surface of the measuring unit so that the effective pressure surface remains the same at all pressures. These measures appear to enable the design of a pressure pickup for use on engine cylinders that combines ample insensitivity to vibrations in pickup axis direction with high natural vibration frequency.

XIV. CONCLUSIONS

Of the conventional methods for suppressing the mass forces in piezoelectric-pickup readings the elastic suspension seems to be the simplest. But the usual pickups with flat-spring suspension, when the springs are used at the same time for the mechanical pre-tension of the quartz, invites the danger of disturbances on account of the nonlinearity characteristics of the springs at conditions encountered in engine cylinders. At the same time, measures are necessary to make the magnitude of the effective part of the pressure-absorbing surface of the pickup independent of the pressure. The natural vibration frequencies of pickups with pressure-transmitting pistons are dependent on the piston size and arrangement even by elastic suspension of the measuring unit. The initial tension of the quartz due to the suspension springs is particularly unfavorable on such pickups because of the greater masses involved. Suggestions for the removal of the described disturbances are indicated.

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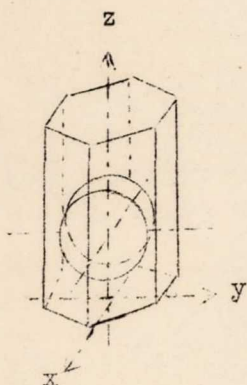


Figure 1.- Orientation of the quartz plate in the crystal.

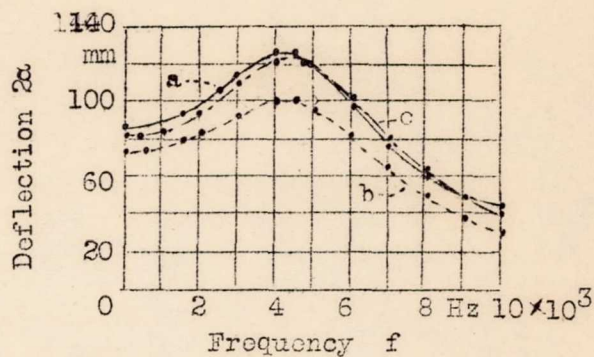


Figure 2.- Frequency of three measuring loops a—b, — — c—natural oscillation characteristics in air 10000 H₂.

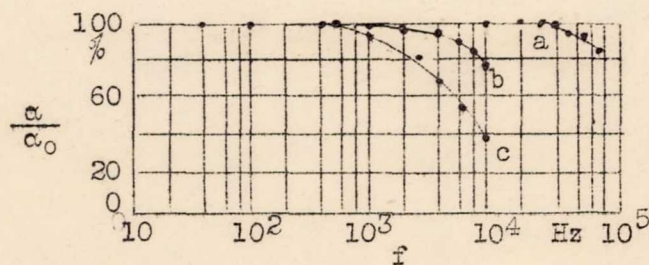


Figure 4.- Frequency of amplifiers of special (a) and commercial (b,c) bands. α_0 = static deflection.

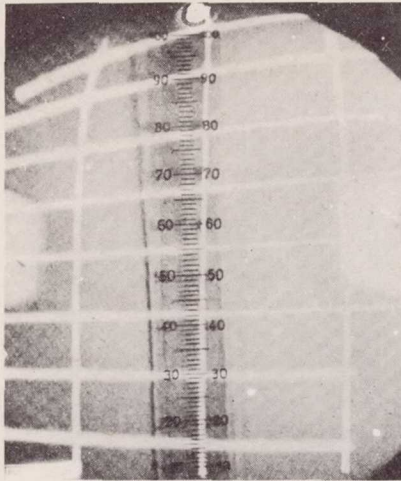


Figure 3.— Coordinate distortion of fluorescent screen of a commercial engine indicator.

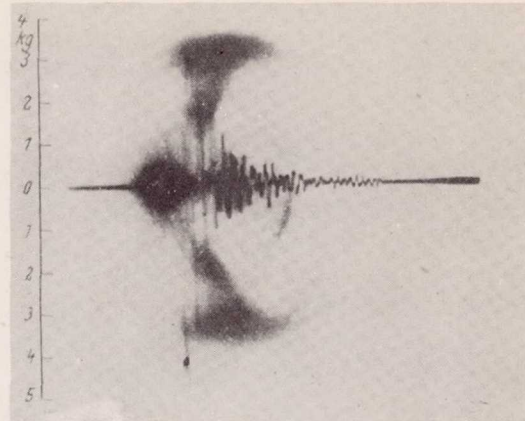
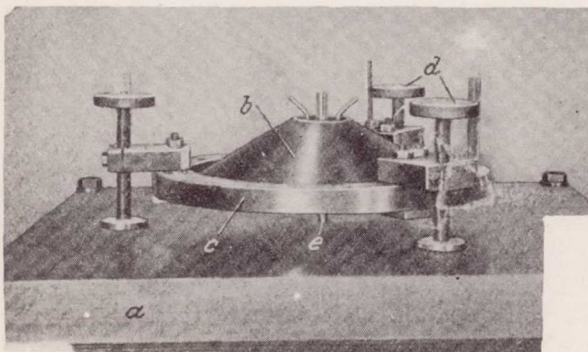
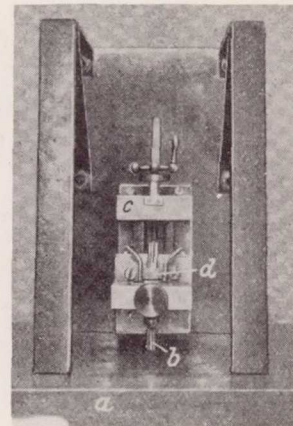


Figure 8.— Free oscillation curve of pickup of fig. 12.

Figure 19.— Steel plate with test electrode and support.
a, Steel plate
b, Cylinder with electrode
c, Support
d, Piezoelectric pickup



a, Steel plate as in fig.19
b. Mass
c. Locking ring
d. Adjusting spindles

Figure 20.— Mass of about 3.6 kg divided over locking ring with three adjusting spindles.

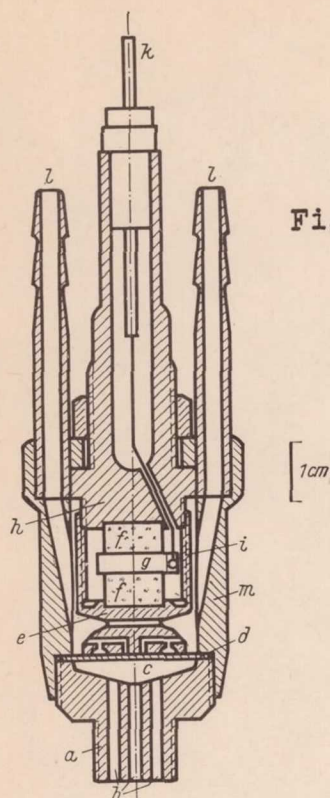
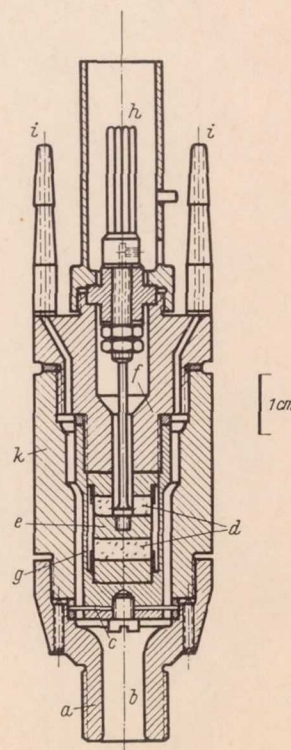


Figure 5.— Commercial piezoelectric pickup with subdivided gas passage.

- a, Spark plug thread
- b, Gas passages
- c, Resonator space
- d, Gas sealing membrane
- e, Pressure transmitting plunger
- f, Quartz
- g, Central electrode
- h, Thrust block
- i, Spring bushing
- k, Heavily insulated contact pin
- l, Cooling water tube
- m, Casing

Figure 6.— Meurer type pickup (our own manufacture).

- a, Spark plug thread
- b, Gas passage
- c, Gas sealing membrane
- d, Quartz
- e, Central electrode
- f, Thrust block
- g, Spring bushing
- h, Heavily insulated contact pin
- i, Cooling water tube
- k, Casing



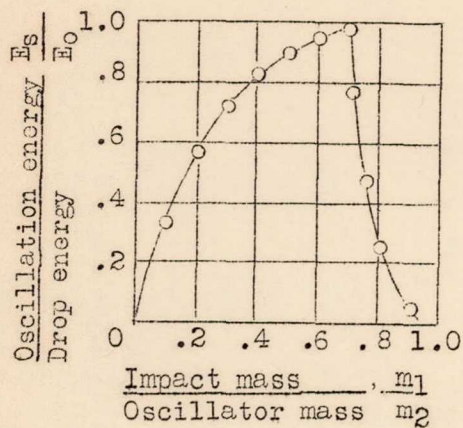


Figure 7.- Ratio of oscillation energy E_s transmitted to an oscillator mass m_2 to drop energy E_o of impact mass m_1 plotted against the ratio of both masses.

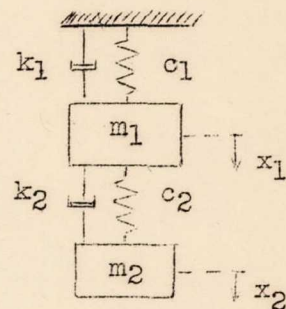


Figure 9.- Simplified substitute view of piezo-electric pickup.

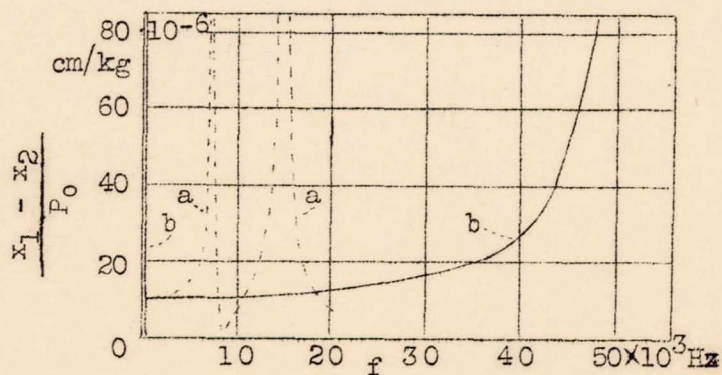


Figure 10.- Resonance curves computed according to equation 12.

Curve a $f_1 = 8,600$ Hz Curve b $f_1 = 500$ Hz

$f_2 = 18,200$ Hz

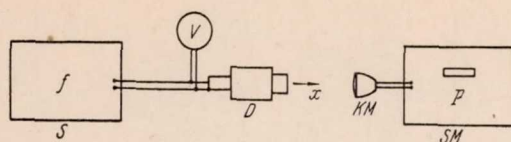
$f_2 = 50,000$ Hz

$\mu_1 = 1:4$

$\mu_1 = 1:100$

$m_2 = 16.3 \cdot 10^{-6}$ kg s²/cm

$m_2 = 10^{-6}$ kg s²/cm



- S, Buzzer with variable frequency f
 D, Pressure pickup
 KM, Condenser microphone
 SM, Sound pressure meter
 V, Voltmeter
 p, Sound pressure

Figure 11.— Experimental sound pressure meter setup.

Figure 12.— Old piezoelectric pickup. model A.

- a, Quartz
 b, Central electrode
 c, Gas sealing membrane and pre-tension spring
 d, Pressure transmitting piston
 e, Casing
 f, Spark plug thread
 g, Thrust block

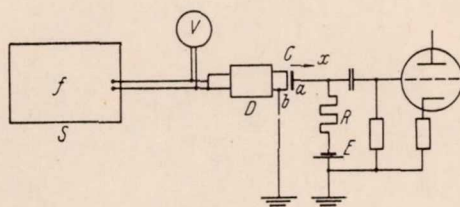
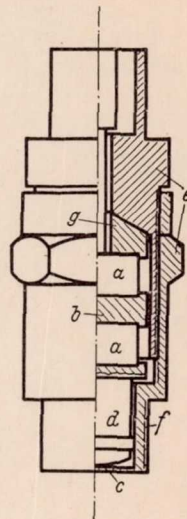


Figure 15.— Modified test method.

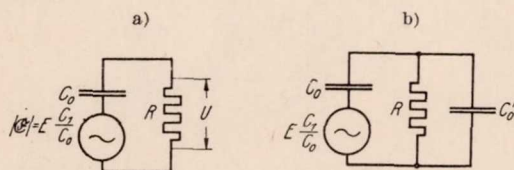


Figure 17.— Substitute diagrams for fig. 15.

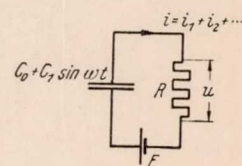


Figure 16.— Hookup for fig. 15.

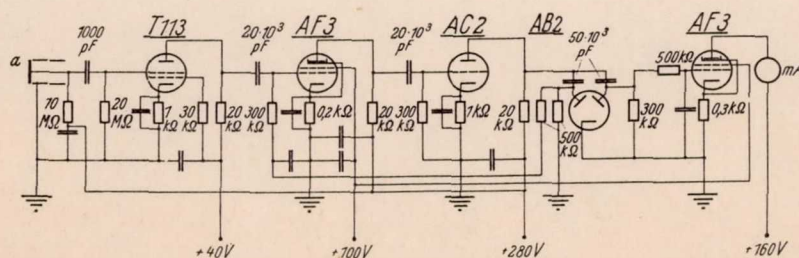


Figure 18.— Wiring diagram of amplitude recorder.

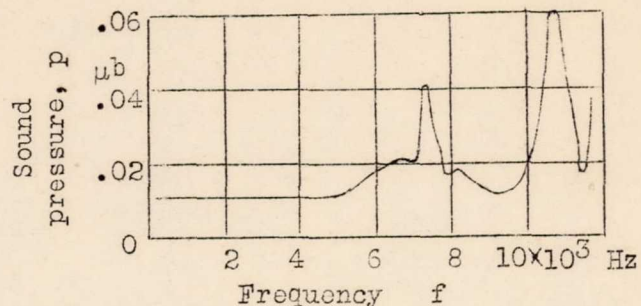


Figure 13.- Resonance curve of pickup of figure 12 recorded with sound pressure meter. Pickup and condenser microphone spaced 56 mm apart; pickup flexibly supported.

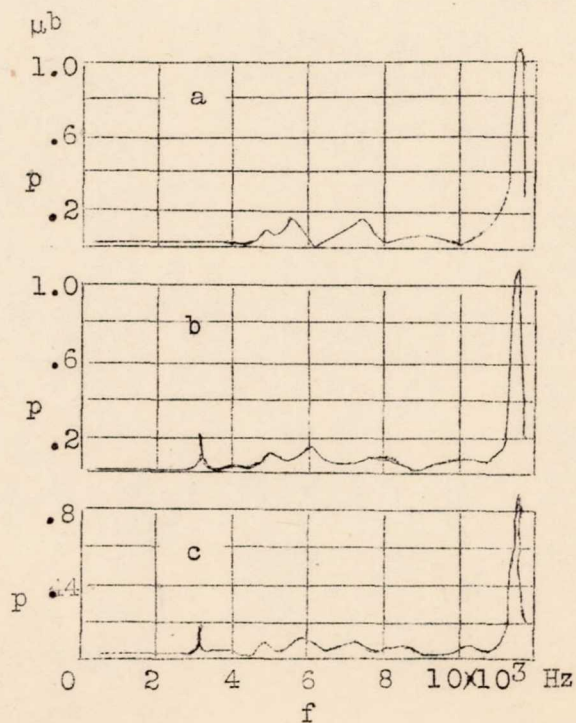


Figure 14.- Resonance curve of pickup of figure 12 recorded with sound pressure meter. Pickup screwed in steel plate.

Curve a Distance 100 mm

Curve b Distance 120 mm

Curve c Distance 140 mm

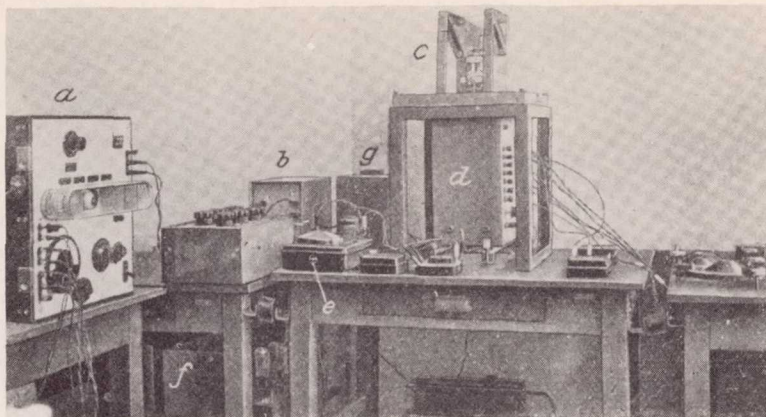


Figure 21.— Experimental layout.

- a, Buzzer
- b, Booster amplifier stage
- c, Steel plate with electrode and support
- d, Amplitude recorder
- e, Reading instrument
- f, Power supply
- g, Electrostatic voltage indicator

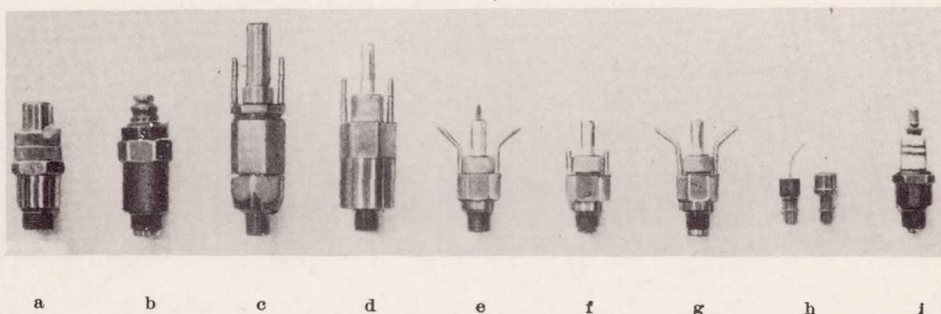


Figure 25.— Experimental specimens.

- a, Pressure pickup model A
- b, Pressure pickup model B
- c, Pressure pickup Meurer (special design)
- d, Piezoelectric pickup with pre-tension flat spring
- e, Piezoelectric pickup. special design with welded spring bushing
- f, Piezoelectric pickup. special design with threaded bushing
- g, Piezoelectric pickup. special design with bottom cap
- h, Welded and threaded spring bushing
- i, Spark plug 14 x 1.25 mm (contrast of size)

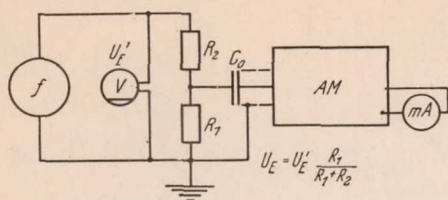


Figure 22.— Calibration hookup for amplitude recorder.

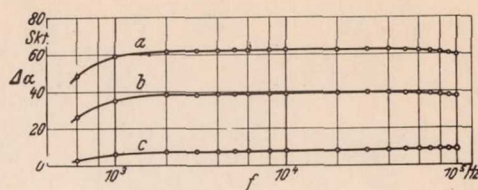


Figure 23.— Frequencies of amplitude recorder at three different amplitudes a. b. c.

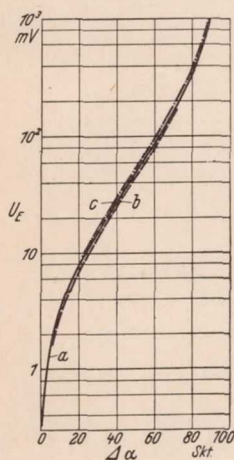


Figure 24.— Calibration curves for amplitude recorder.

- a $f = 2 \text{ kHz}$
- b $f = 10 \text{ kHz}$
- c $f = 100 \text{ kHz}$

- a, Spark plug thread
- b, Pressure transmitting piston
- c, Gas sealing membrane
- d, Pre-tension flat spring
- e, Quartz
- f, Thrust block
- g, Highly insulated contact pin
- h, Cooling water tube
- i, Casing

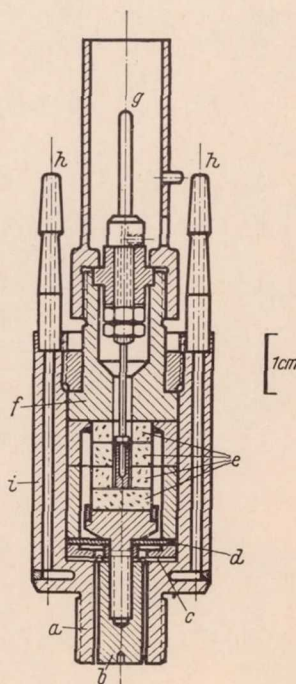


Figure 26.— Pickup with pre-tension flat spring (special design).

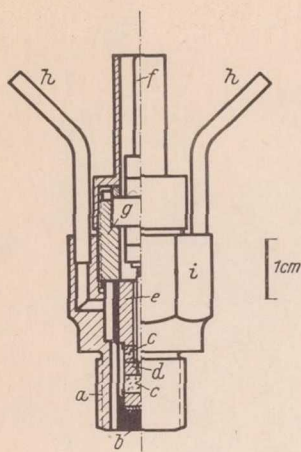


Figure 27.— Pickup, special design, welded spring bushing.

- a, Spark plug thread
- b, Spring bushing
- c, Quartz
- d, Central electrode
- e, Thrust block, welded into spring bushing
- f, Heavily insulated contact pin
- g, Thumb screw
- h, Cooling water tube
- i, Casing

Figure 28.— Pickup, special design, with threaded spring bushing and bottom cap.

- a, Spark plug thread
- b, Bottom screw, gas sealing membrane welded
- c, Bottom cap
- d, Spring bushing
- e, Quartz
- f, Central electrode
- g, Thrust block, screwed in spring bushing
- h, Heavily insulated contact pin
- i, Thumb screw
- k, Cooling water tube
- l, Cooling water chamber
- m, Casing

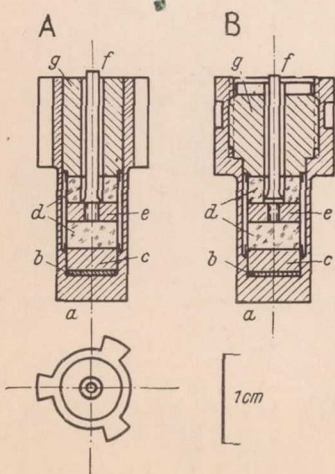
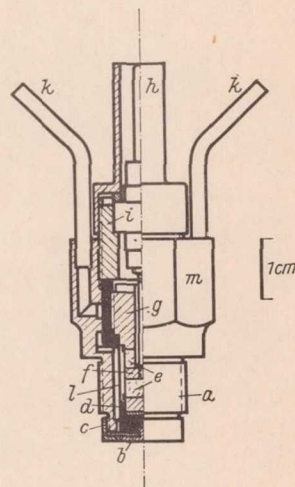


Figure 29.— Spring bushings of pickups of special design.

- A, Welded
- B, Threaded
- a, Bushing bottom
- b, Aluminum leaf
- c, Pressed-in electrode
- d, Quartz
- e, Central electrode
- f, Heavily insulated contact pin
- g, Thrust block

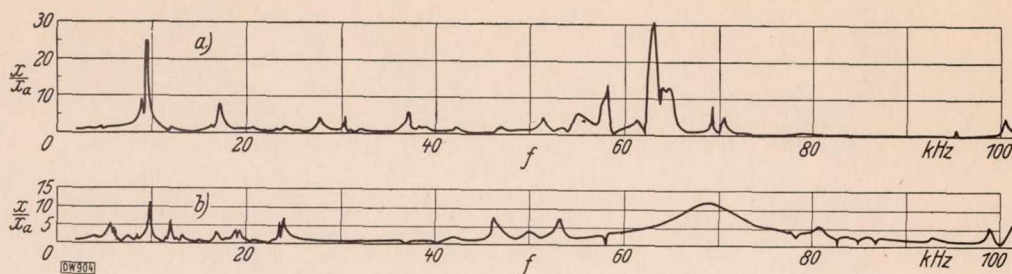


Figure 30.- Resonance curves of pickup fig. 12, model A.
a, Without b, With mass

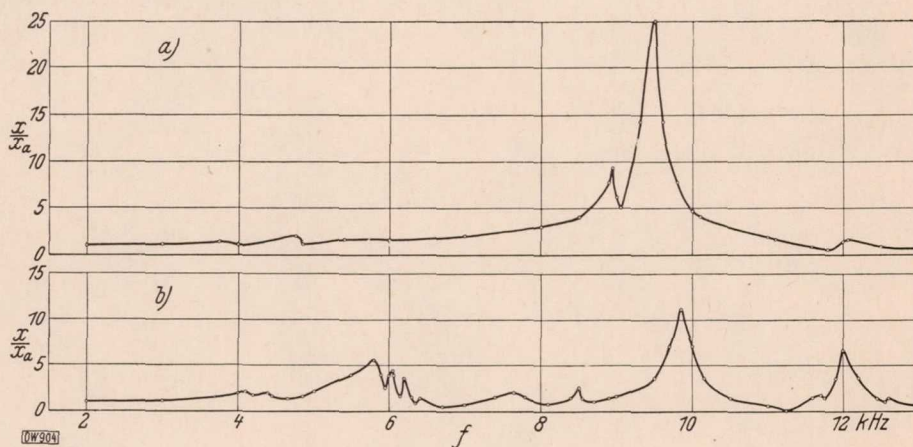


Figure 31.- Magnified section of fig. 30, showing test points.

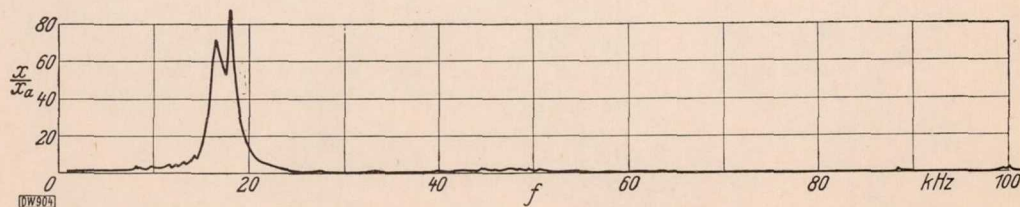


Figure 32.- Resonance curves of pickup, model B.

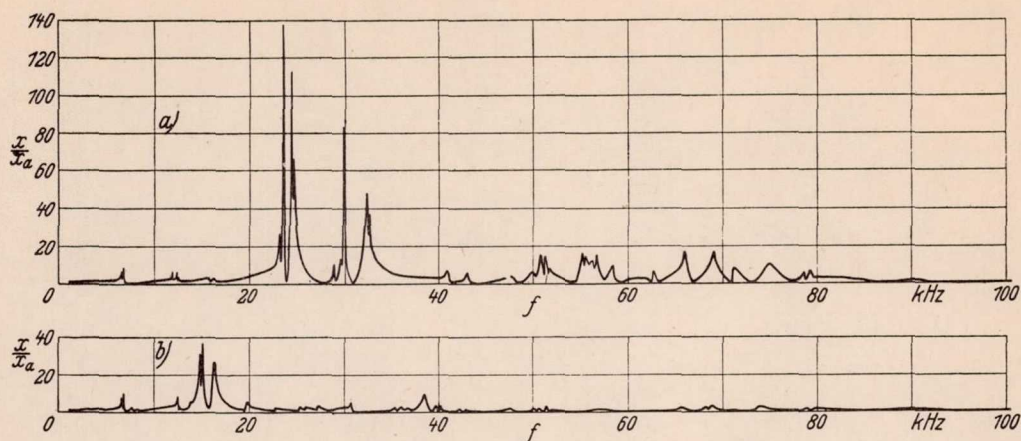


Figure 33.- Resonance curves of pickup, Meurer type.
a, With mass
b, With mass and additional mass before quartz.

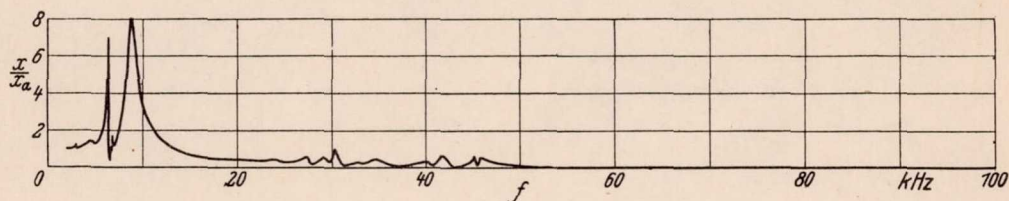


Figure 34.- Resonance curve of pickup with pre-tension flat spring, with mass.

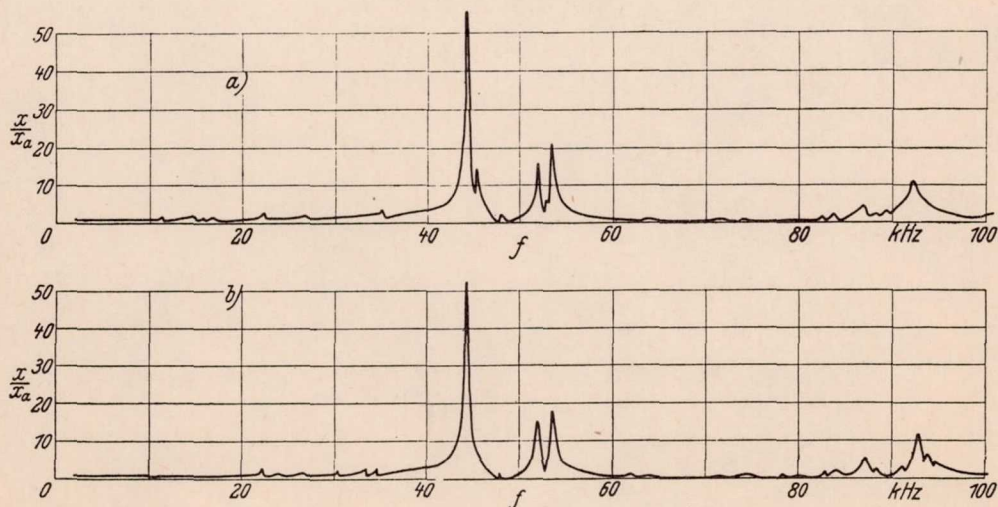


Figure 35.- Resonance curves of pickup of special design, welded bushing.
a, With mass
b, With mass and enlarged plug mass

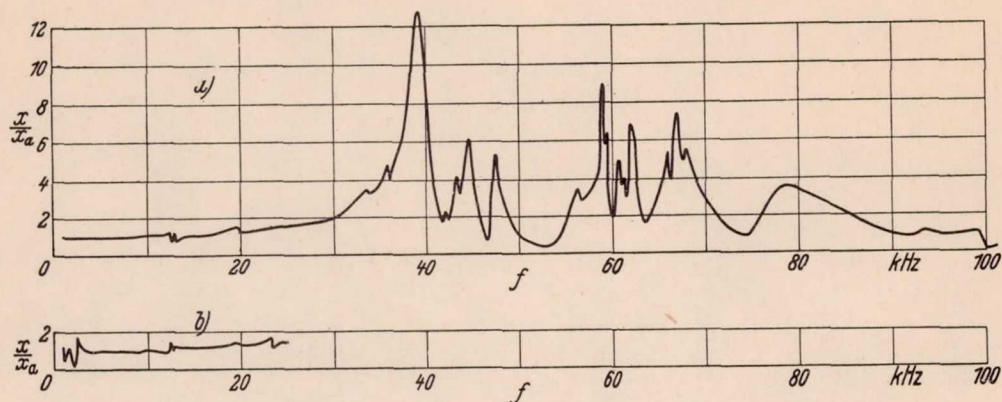


Figure 36.- Resonance curves of pickup of special design with screwed on bushing, without mass.
 a, Plug pin uncoupled
 b, Plug pin solidly connected

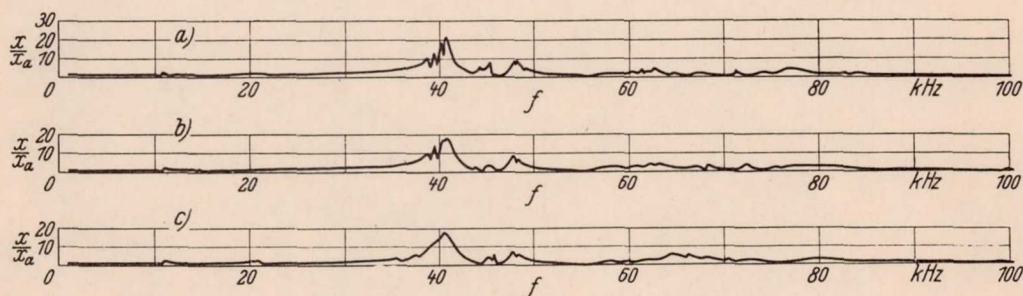


Figure 37.- Resonance curves of pickup of special design with screwed on bushing.
 a, Mass of 3.6 kg
 b, Mass of 0.73 kg
 c, Mass of 0.73 kg plus enlarged plug mass

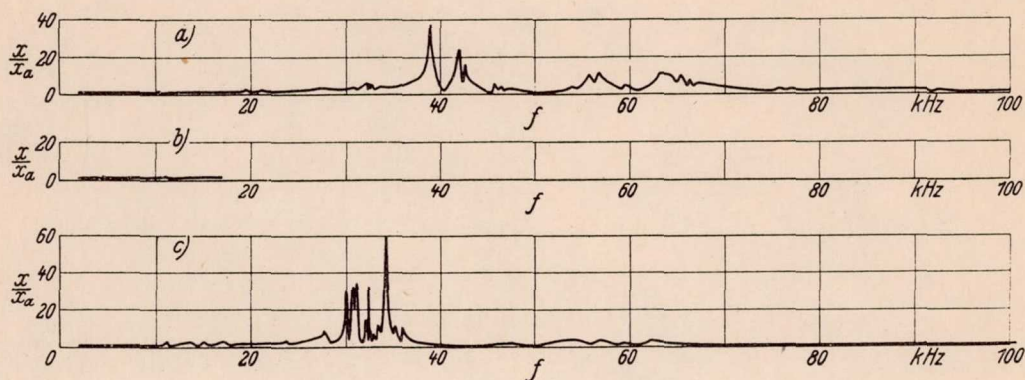


Figure 38.- Resonance curves of pickup, special design, screwed on spring bushing with bottom cap.
 a, With mass, iron-asbestos washer
 b, With mass, aluminum washer
 c, With mass, iron-asbestos washer and additional mass before Quartz

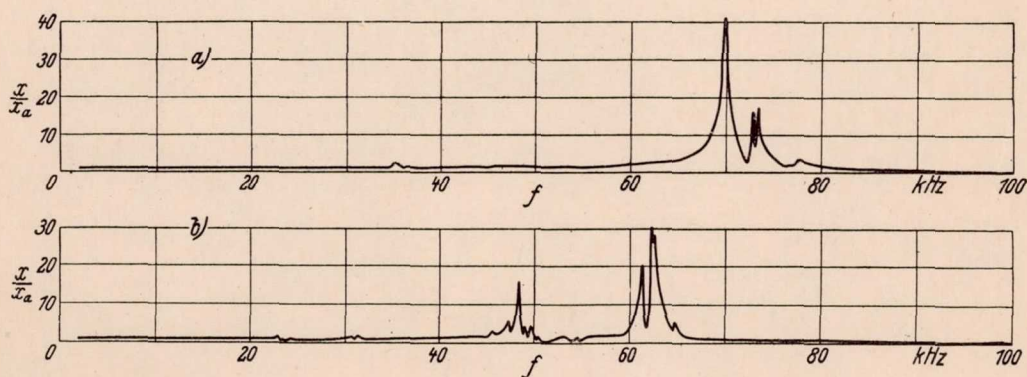
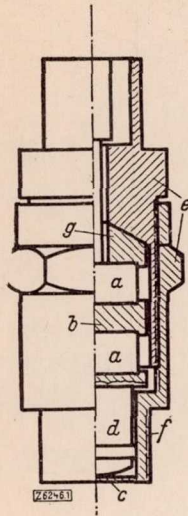
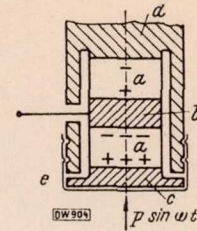


Figure 39. Resonance curves of spring bushings.
 a, Welded
 b, Screwed on } according to fig. 29



- a, quartz crystals
- b, central electrode
- c, pre-tension spring
- d, pressure transducing piston
- e, casing
- f, spark plug thread
- g, thrust block

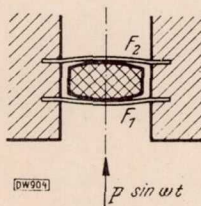
Figure 40.- Pressure pickup-old model.



- a, quartz
- b, central electrode
- c, cover plate
- d, casing
- e, closing foil

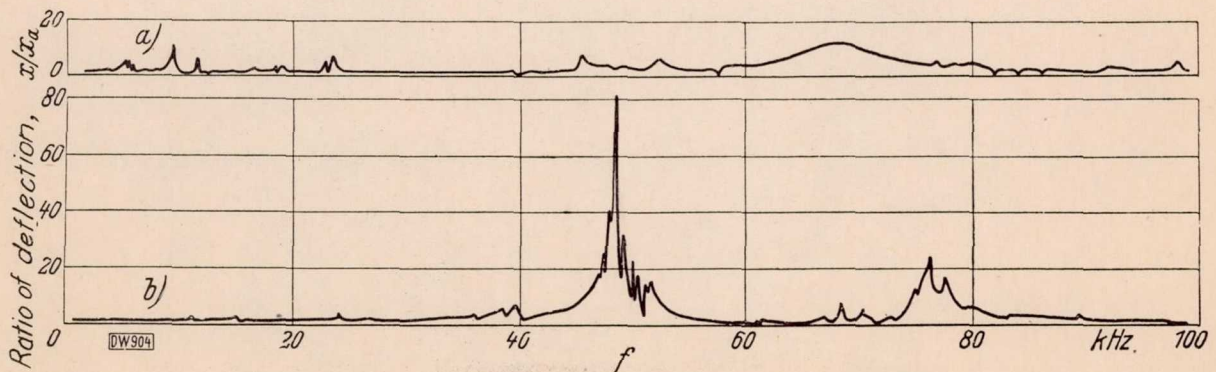
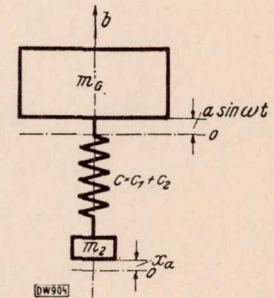
Figure 41.- Pressure pickup with quartz of different piezo-electric modulus used for acoustic measurements.

Figure 42.- Pressure pickup with quartz suspended under initial tension between two flat springs F_1 and F_2 .



- m_G , casing mass
- m_2 , mass of quartz and springs

Figure 43.- Substitute layout of pickup, figure 42.



a, old model

b, new model

Figure 44. Resonance curves of piezoelectric pickup with rigid thrust block.

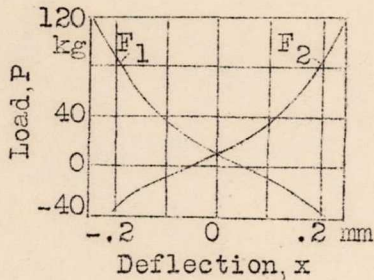


Figure 45.- Computed spring characteristics of F_1 and F_2 of pickup, Figure 3.

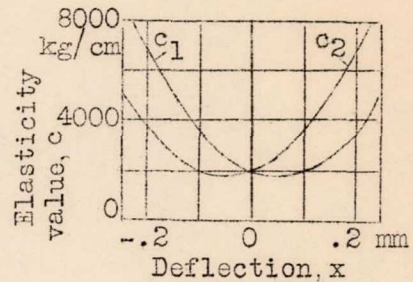


Figure 46.- Computed elasticity values c_1, c_2 of springs F_1, F_2 of pickup of Figure 3 for different deflection x .

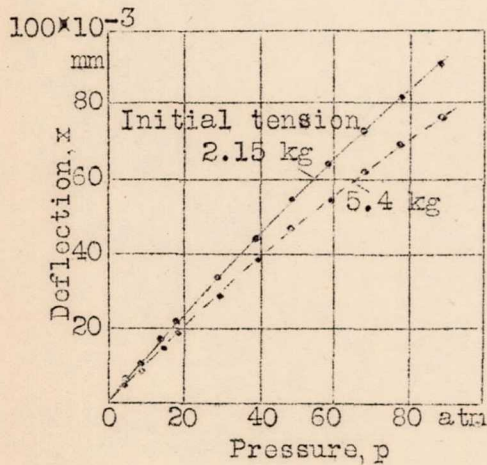


Figure 50.- Experimental deflection x of system F_1+F_2 under variable pressure p at two different initial tensions of the springs.

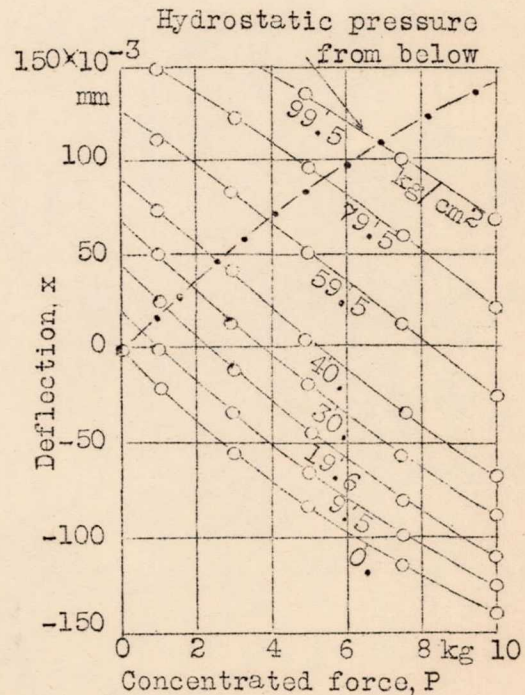
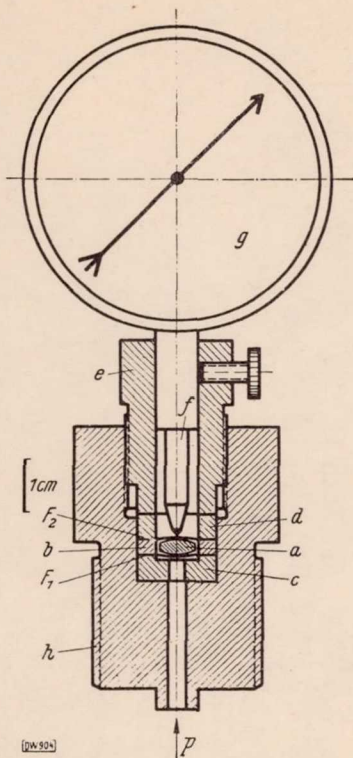


Figure 51.- Experimentally defined deflection x of bottom spring F_1 under variable concentrated load P from above at different loads with moderate hydrostatic pressure p from below. The -.- curve represents the experimentally defined deflection x of the top spring F_2 under various concentrated loads P from below.



- a, lens shaped steel element
- b, spacer
- c,d, clamping rings
- e, pressure screw
- f, feeler
- g, dial gauge
- h, threaded nipple for oil pressure head
- F₁, F₂, spring steel shim

Figure 47.- Experimental setup for loading the measuring system with oil pressure.

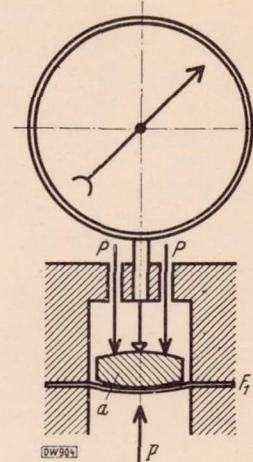


Figure 48.- Simultaneous loading of bottom spring F₁ by oil pressure p from below and concentrated load P from above.

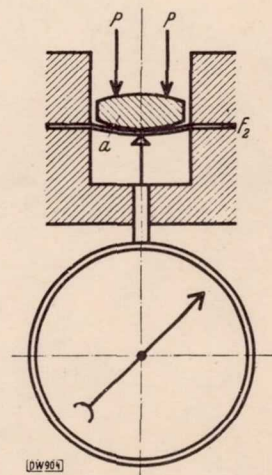


Figure 49.- Loading of top spring F₂ under concentrated load P.

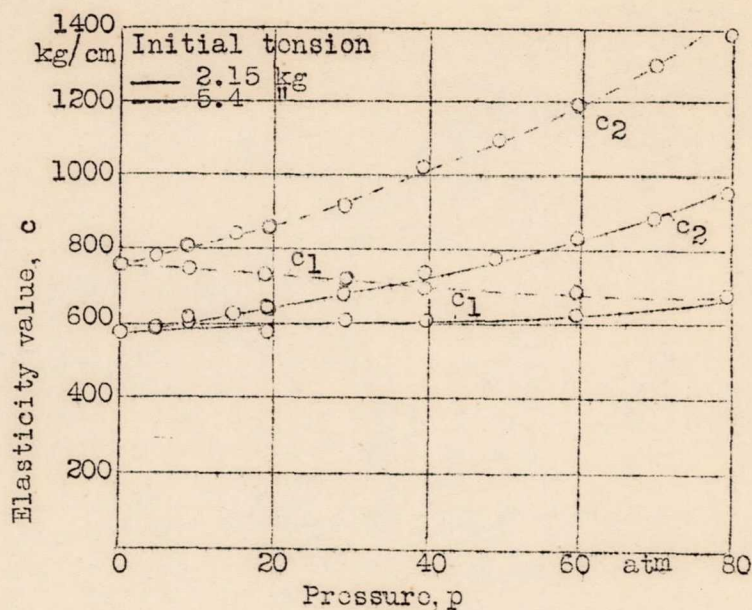


Figure 52.- Experimental elasticity values c_1, c_2 of springs F_1, F_2 at 2.15 and 5.4 kg initial tension.

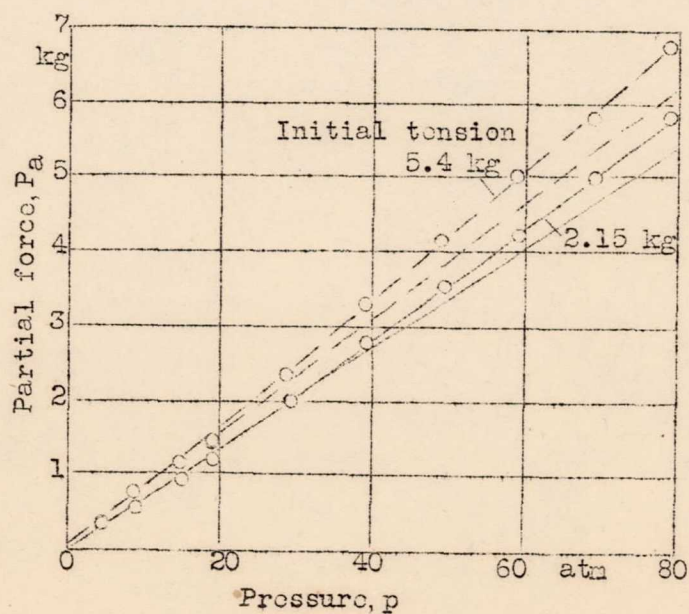


Figure 53.- Portion of force P_a indicated by quartz under variable pressure p at 2.15 and 5.4 kg initial tension.

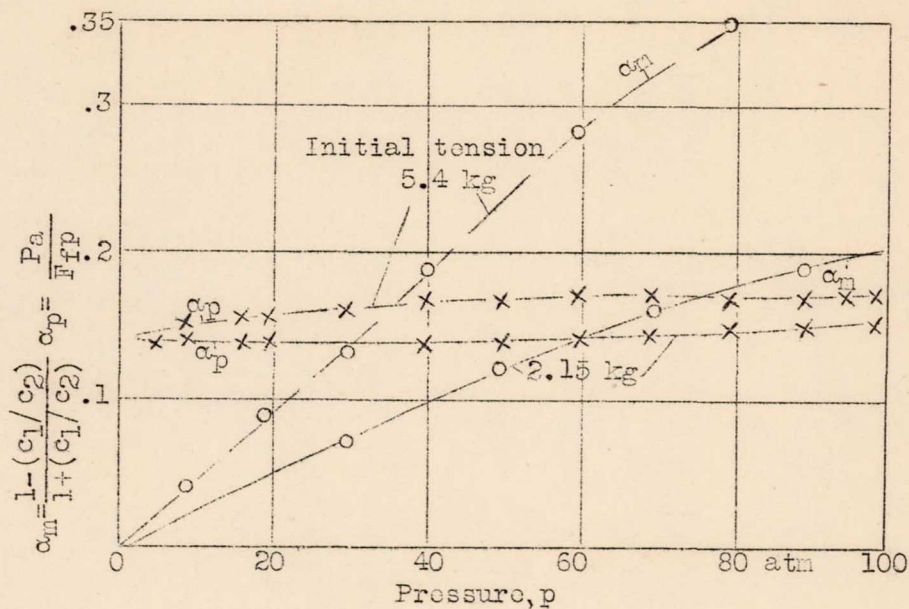


Figure 54.- Mass force factor α_m and pressure force factor α_p , determined from the experimental values at 2.15 and 5.4 kg initial tension.

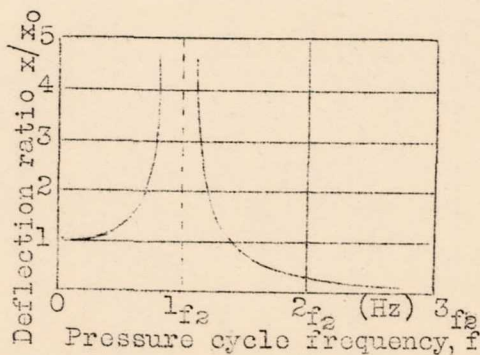
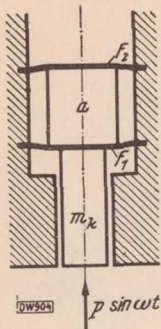


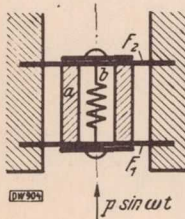
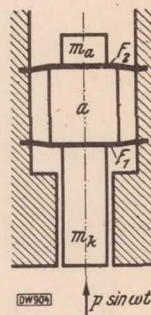
Figure 55.- Ratio of deflection x/x_0 of quartz at variable pressure frequency f x_0 static deflection; (substitute diagram 4).



a, quartz
F1, F2, flat springs

Figure 56.- Quartz with elastic suspension and pressure transmitting piston m_k .

Figure 57.- Same as figure 56, but supplemented by balancing mass m_a .



a, quartz
b, tension spring
F1, F2, flat springs

Figure 58.- Arrangement of supplementary pre-tension device for relieving flat springs

a, quartz
b, tubular spring, water cooled
c, pressure linkage
d, counter mass
e, flexible gas sealing membrane
f, cooling water chamber
F1, F2, unloaded flat springs

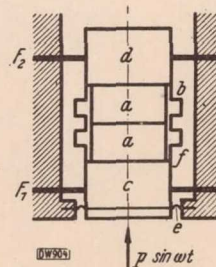


Figure 59.- Avoidance of pressure regulation by gas sealing membrane with minimum annular clearance.